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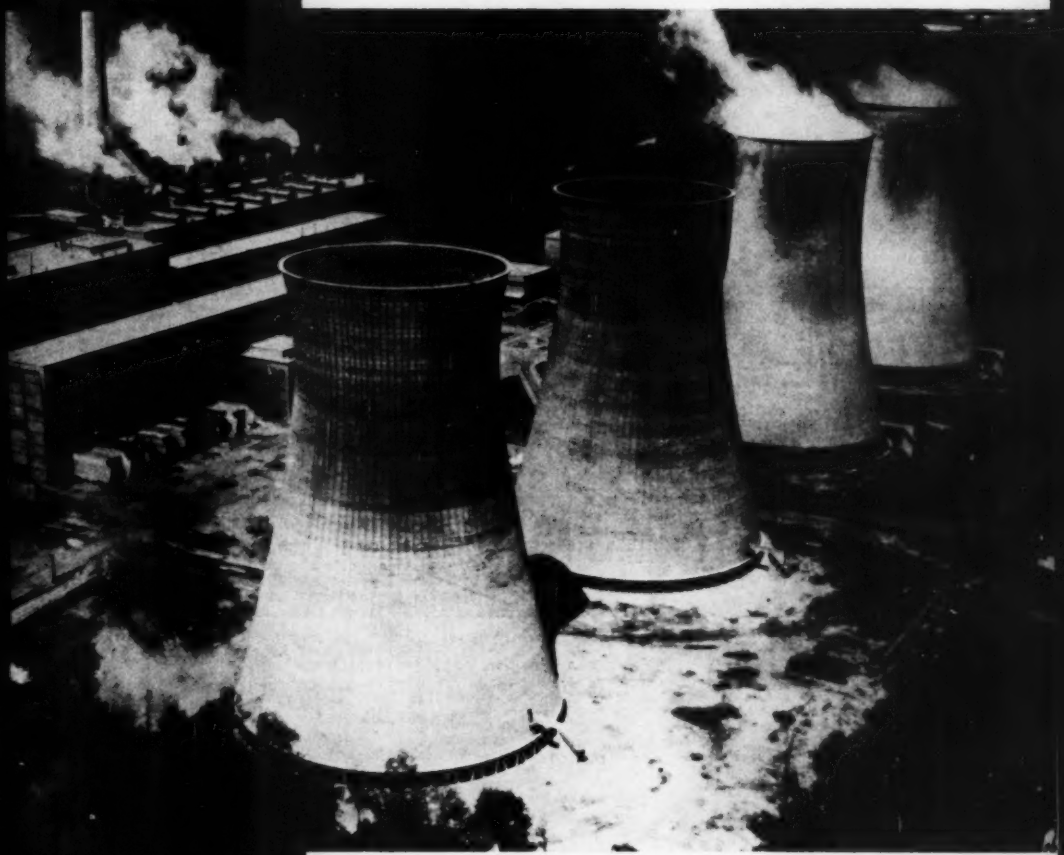

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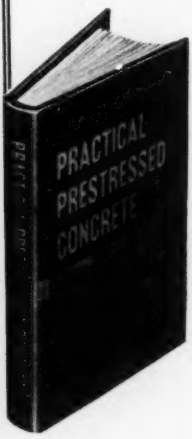

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On the Cover

Four massive yet graceful looking hyperbolic reinforced concrete cooling towers in Lorraine, France. The two towers in the foreground have only recently been erected and each will have a capacity of 92,500 gal per min in full operation. Read Paul Rogers' informative paper on reinforced concrete cooling towers starting on p. 395.—Photo courtesy Hamon Co., Brussels.

Coming next month in the JOURNAL

ACI COMMITTEE 621, Aggregates, reports on "The Selection and Use of Aggregates for Concrete." The report is limited to sand, gravel, crushed stone, and air-cooled blast-furnace slag.

•

W. JOHN HUFSCHMIDT describes the design, casting, and erection of "Precast Complex Conoidal Horticultural Domes," at Milwaukee's Botanical Garden Development at Mitchell Park.

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B. BRESLER and P. GILBERT discuss the mechanism of failure of R/C columns with lateral ties in their paper, "Tie Requirements for Reinforced Concrete Columns."

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EMILIO ROSENBLUETH and ROGER DIAZ DE COSSIO present a photographic study of "Reinforced Concrete Failures During Earthquakes," highlighting major types of damage.

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M. F. KAPLAN describes analytical and experimental procedures used to predict "Crack Propagation and the Fracture of Concrete."

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Results of research on the "Influence of Sand Concentration on the Deformations of Mortar Beams under Low Stresses," is reviewed by O. ISHAI.

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JOHN J. REED and C. DAVID MANN detail an effort to develop stronger, more effective concrete pillars for underground mine support in their paper, "Full Scale Testing Develops Efficient Preloaded Concrete Pillars."

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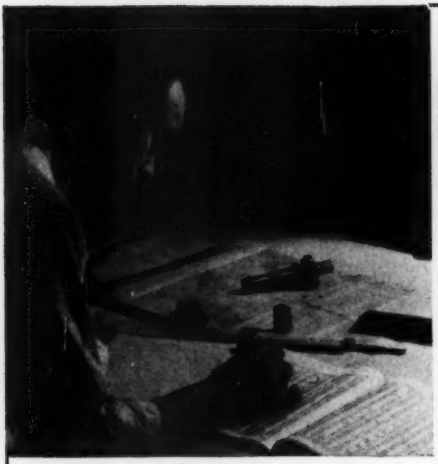

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Slabless Tread-Riser Stairs

By LUIS P. SAÉNZ and IGNACIO MARTÍN

The elastic analysis of orthopolygonal stairways in a plane, fixed at both ends, is presented by two different methods. The first method is based on column analogy and Newmark's numerical procedure for computing shears and moments. The second method is based on the determination of the constant second differences of the general expression of the end moments and then, by exact extrapolation, the moment for any number of steps can be obtained. The design of this type of stairway is also discussed.

■ A PLANE ORTHOPOLYGONAL STRUCTURE is one whose axis lies within a plane and consists of a continuous broken line whose segments form angles of approximately 90 deg between them (see Fig. 2).

The construction of some stairways of this type, with approximately plane orthopolygonal axes, has stirred the interest of many architects and engineers. The publication of data on the design of these structures should therefore be welcome. This paper explains two of the methods that can be used for designing structures of this type:

I — Column analogy method
II — Second differences method

STATEMENT OF PROBLEM

Before describing the methods in detail, it is proper to explain the general simplification which may be introduced in either one of the two methods.

As is well known, the slope deflection method has the advantage of extensive application. Knowing the material, the transverse section of the member and its span, the value of the end moments of the member depend on the load supported by it, the rotation of the tangents at the ends of the axis of the member, and the displacement of one end of the member with respect to the other.

If the axis of a member with constant section is originally AB' (see Fig. 1) and, after loading, its end A has rotated through an angle θ_A, its end B through an angle θ_B, and it has been displaced an amount d, and E represents the longitudinal modulus of elasticity of the material, I the moment of inertia of the transverse section with respect to the

ACI members **Luis P. Saenz** and **Ignacio Martin** are partners in the engineer-architect firm of Saenz-Cancio-Martin-Gutierrez (SACMAG), in Havana, Cuba, and San Juan, Puerto Rico, respectively. Both have previously taught at the University of Havana and have been closely associated with a wide variety of engineering projects in Cuba, Puerto Rico, and Mexico.

neutral plane, and L the span; the end moments of Member AB are expressed by the following equations:

$$M_{AB} = M_{FAB} + \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3d}{L}\right) \quad (1)$$

$$M_{BA} = M_{FBA} + \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3d}{L}\right) \quad (2)$$

where

M_{FAB} = fixed end moment at End A due to the load acting on Span AB'
M_{FBA} = fixed end moment at End B due to the load acting on Span AB'

Fig. 2 shows a number of steps of a stairway with a plane orthopolygonal axis. The dead load consists of the weight of the steps and the weight of the material superposed or cover, if any. The live load is the weight imposed by transit on the stairway, which is generally supposed to be uniformly distributed. Both loads are shown in Fig. 2.

Stating the problem in this way makes the solution of Eq. (1) and (2) difficult for each tread and each riser. The statement can be simplified as shown in Fig. 3 without practical loss of accuracy. In fact, by concentrating the dead and live loads on the axis of the riser, we can eliminate the end moments from Eq. (1) and (2). The substitution of the distributed load by a concentrated one is reasonable, because the effect of the dead load of the tread on the span L is small and the live load bears down principally on the riser as indicated in Fig. 3. With all risers and treads equal, Eq. (1) and (2) are reduced to

$$M_{AB} = \frac{2EI}{L}\left(2\theta_A + \theta_B - \frac{3d}{L}\right) \quad (3)$$

$$M_{BA} = \frac{2EI}{L}\left(2\theta_B + \theta_A - \frac{3d}{L}\right) \quad (4)$$

METHOD I — COLUMN ANALOGY

The column analogy method of Cross[1] is based on the mathematical identity between the stresses produced in a short column and the moments in a framed structure.

It is convenient to use this method for the design of stairways with orthopolygonal axes because

1. The centroid of the analogous column can be found at once, since these are generally symmetrical structures
2. The determination of the properties of just two elements, tread and riser, is sufficient to obtain the data required for applying the method

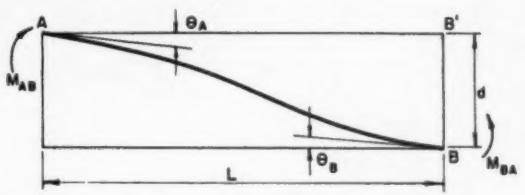


Fig. 1—Restrained member after displacement of one end, indicating rotation of the end tangents

Newmark's numerical procedure for computing isostatic bending moments

This numerical procedure[2] furnishes a quick way of computing the isostatic bending moments and is based on the principle of statics: "that the shear in a section of a beam subjected to flexure is equal to the algebraic sum of the loads on one side of that section, and the bending moment at the same section is the algebraic sum of the shears multiplied by the length of the beam upon which they act."

The reasons for employing this method for determining the isostatic moments in the stairways are:

1. The treads and risers usually have constant dimensions in a stairway, so that the lengths of application of the shear can be maintained as a common factor in the computations

2. This procedure can be extended to find the area of the bending moment diagram, which divided by the product of the moment of inertia and the longitudinal modulus of elasticity will give the load of the analogous column

METHOD II — SECOND DIFFERENCES

At first, this method follows the general solution of slope deflection, but has the inconvenience that when the number of unknowns is large, the number of equations required makes the solution laborious, so the pattern of the slope deflection method is used to determine the second differences of the general expression of the end moments with great accuracy; which then by exact extrapolation can be used for any number of steps in the case of stairways.

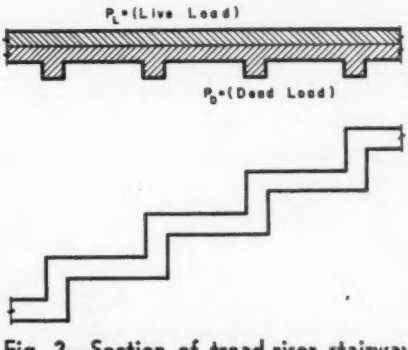


Fig. 2—Section of tread-riser stairway showing loading

Equations of equilibrium and symmetry

To determine the value of the end moments of the members it is necessary to satisfy certain conditions of equilibrium and symmetry. In this case the condition of equilibrium for the joints, e.g., Joint B in Fig. 3, requires $\Sigma M_B = 0$.

The condition of equilibrium for the bars, e.g., Bar AB, requires

$$\frac{M_{AB} + M_{BA}}{L_{AB}} + R_A = 0$$

The condition of symmetry, for the bars, e.g., Bar EE', requires

$$\theta_E + \theta_{E'} = 0$$

This holds true for cases with an odd number of steps. In cases with an even number of steps, the condition of symmetry is identical with the condition of equilibrium for a joint.

Such conditions have to be fulfilled in any joint or bar for structures of this type. The condition of symmetry cuts the number of unknowns in half. Equilibrium conditions for the joints and bars determine the angles of rotation of the end tangents of the axis of the members, and the horizontal and vertical displacements of the ends of these axes. By forming the corresponding equations the problem is solved theoretically. But using this method when the number of steps exceeds say, five, which is usual in any stairway, the solution of the problem involves the solution of a large number of simultaneous equations. Even with the use of electronic computers now available it requires a considerable amount of time and work, which may lead to mistakes in the calculations.

As an example, let us consider the case shown in Fig. 3, which is a stairway with only five treads and four risers. Fig. 3 is a diagram of the axis of Stairway $ABCDEE'D'C'B'A'$ with ends fixed at A and A'. The deformation of the axis is shown in Fig. 3 (right), which also indicates the rotations and displacements of the assembly of bars and joints that constitute the axis.

The system of equations for solving this case involves nine equations and nine unknowns.

It is not difficult to imagine that if the stairway of five treads shown in Fig. 3 requires a system of nine equations with nine unknowns, the ordinary case of a stairway with 18 treads would require no less than 35 equations with 35 unknowns by following the same method.

To obviate this solution it is worthwhile to note that in the system of equilibrium equations, the unknowns are related to each other in a simple way. It is sufficient to determine just one unknown for solving the entire system. Suppose that for the problem stated it would be possible by some device to determine the end moment M_{AB}. Introducing this value in the proper equation of the system M_{BA} is obtained. Using this value, M_{BC} can be determined. With it, the value of M_{CB} can be obtained and successively all the end moments of the members as well, thus giving an easy solution to the problem. A device has to

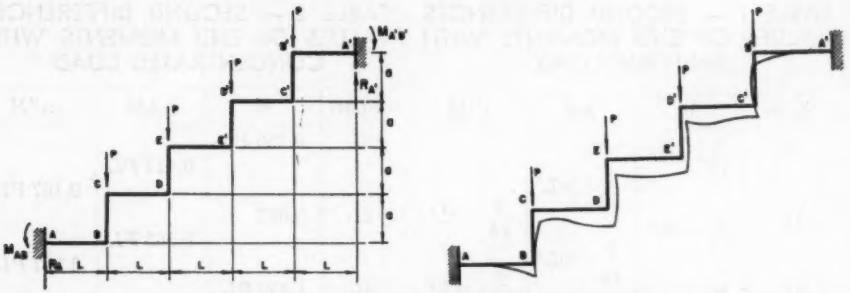


Fig. 3—Axis of tread-riser stairway indicating the simplified loading system (left), and the axis after deformation (right)

be found that can be used for determining the end moments without complications.

Constant value of the second differences in general expression of end moments

It should be noted that each step added to the stairway introduces one tread, one riser, and one load which are constants. Hence, it can be supposed that the increments produced in the end moments due to the progressive increase in the number of steps are subject to a simple law of variation.

The end moments produced by uniformly distributed loads or by numerous equally spaced concentrated loads at constant sections of the span of a member can be expressed by an equation of the following type:

$$y = a + bx + cx^2 \quad \text{.....(5)}$$

The equations of the class of curves in which the abscissa varies in accordance with an arithmetic series, have constant second differences, as can be easily demonstrated.

Let us call the first difference of the ordinate Δy and the second difference $\Delta^{II} y$. In the same way, let Δx be the increment which is to be applied to abscissa x. Incrementing the abscissas we have

$$y + \Delta y = a + b(x + \Delta x) + c(x + \Delta x)^2 \quad \text{.....(6)}$$

Subtracting Eq. (6) from Eq. (5), the first difference is

$$\Delta y = b\Delta x + 2cx\,\Delta x + c(\Delta x)^2 \quad \text{.....(7)}$$

Incrementing this first difference

$$\Delta y + \Delta^{II} y = b\Delta x + 2c(x + \Delta x)\,\Delta x + c(\Delta x)^2 \quad \text{.....(8)}$$

Subtracting Eq. (7) from Eq. (8) the second difference is

$$\Delta^{II} y = 2c(\Delta x)^2 = \text{constant}$$

TABLE 1 — SECOND DIFFERENCES VALUES OF END MOMENTS WITH UNIFORM LOAD

Span	M	ΔM	$\Delta^{II} M$
$1L$	$\frac{1}{12} wL^2$		
		$\frac{3}{12} wL^2$	
$2L$	$\frac{4}{12} wL^2$		$\frac{2}{12} wL^2$
		$\frac{5}{12} wL^2$	
$3L$	$\frac{9}{12} wL^2$		$\frac{2}{12} wL^2$
		$\frac{7}{12} wL^2$	
$4L$	$\frac{16}{12} wL^2$		$\frac{2}{12} wL^2$
		$\frac{9}{12} wL^2$	
$5L$	$\frac{25}{12} wL^2$	.....	.....
$6L$	.....		

TABLE 2 — SECOND DIFFERENCES VALUES OF END MOMENTS WITH CONCENTRATED LOAD

Span	M	ΔM	$\Delta^{II} M$
$2L$	$0.250\ PL$		
		$0.417\ PL$	
$3L$	0.667		$0.167\ PL$
		$0.583\ PL$	
$4L$	$1.250\ PL$		$0.167\ PL$
		$0.750\ PL$	
$5L$	$2.000\ PL$		$0.167\ PL$
		$0.917\ PL$	
$6L$	$2.917\ PL$	.....	.....
	.....		
...			

Since Δ_x is constant, it represents the difference between the magnitudes that form the arithmetic series of the abscissas.

The above can easily be checked with a few ordinary examples. Let us take the case of a fixed end beam of span L subjected to a uniformly distributed load and consider successively the cases of a span of $2L$, $3L$, $4L$, etc. The value of the second differences of the end moments can be calculated as shown in Table 1.

Let us now suppose the case of a fixed end beam with a concentrated load P at the center of span $2L$, and let us increase the span successively to $3L$, $4L$, $5L$, etc., adding a concentrated load P at L distance. The value of the second difference of the end moments can be calculated as shown in Table 2.

To determine the constant second difference it is only necessary to calculate three values of the function which simplifies the problem considerably.

The formula expressing the n value of the ordinate y as a function of the first term of the arithmetic series and of its increments can be obtained as follows:

$$y_1 = y_1$$

$$y_2 = y_1 + \Delta y_1$$

$$y_3 = y_2 + \Delta y_2 = y_1 + \Delta y_1 + \Delta(y_1 + \Delta y_1) = y_1 + 2\Delta y_1 + \Delta^{II} y_1$$

$$y_4 = y_3 + \Delta y_3 = y_1 + 3\Delta y_1 + 3\Delta^{II} y_1 + \Delta^{III} y_1$$

The coefficients of these expressions follow the law of the coefficients of the Binomial Theorem, that can be written in the following way:

$$y_n = y_1(n-1)\,\Delta y_1 + \frac{(n-1)(n-2)}{1\times 2}\,\Delta^{II}y_1 + \frac{(n-1)(n-2)(n-3)}{1\times 2\times 3}\Delta^{III}y_1 + \ldots\ldots\ldots\ldots\ldots\ldots \text{etc.}$$

In our case the second difference is a constant and for that reason the third and following differences are equal to zero. The formula is reduced to

$$y_n = y_1 + (n-1)\Delta y_1 + \frac{(n-1)(n-2)}{2}\,\Delta^{II}y_1$$

Then the exact extrapolation for any number of steps which the stairway may have, can be easily calculated also, knowing the first term and its differences by using the preceeding formula:

$$M_n = M_1 + (n-1)\Delta M_1 + \frac{(n-1)(n-2)}{2}\,\Delta^{II}M_1 \quad \ldots\ldots\ldots\ldots (9)$$

where

M_n = fixed end moment of stairway
M_1 = fixed end moment for case of one riser
n = number of risers
ΔM_1 = difference between fixed end moments with one and with two risers
$\Delta^{II}M_1$ = second constant difference of the variation of the fixed end moments

For example in the case of a fixed end beam with a uniformly distributed load and a span of $5L$, M_5 would be

$$M_5 = \frac{wL^2}{12} + \frac{4\times 3\,wL^2}{12} + \frac{4\times 3\times 2\,wL^2}{2\quad 12} = \frac{25\,wL^2}{12}$$

which is the value found previously.

With the methods explained here it is possible to design a stairway with a plane orthopolygonal axis and any number of steps within a reasonable time.

EXAMPLE

Let us consider the case of an integral terrazzo stairway with the following characteristics (see Fig. 4).

Loads

Tread slab dead load $= \dfrac{4\times(11+5)\,40\times 150}{(12)^2} = 222$ lb

Riser slab dead load $= \dfrac{5\times 3\times 40\times 150}{(12)^2} = 52$ lb

Dead load = 274 lb

Live load $= \dfrac{11\times 40\times 100}{(12)^2} = 306$ lb

Total load = 580 lb

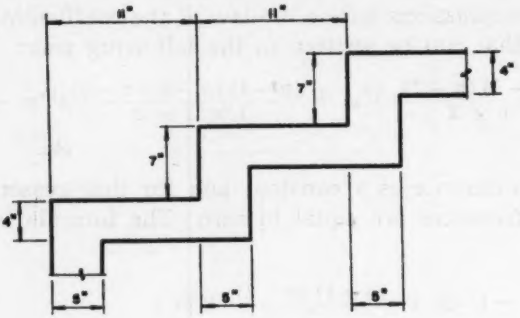


Fig. 4—Dimensions of tread-riser stairway

No. of treads = 12
No. of risers = 11
Stairway width = 40 in.

Constants

$$I_h = \frac{40 \times 4^3}{12} = 213.33 \text{ in.}^4$$

$$I_v = \frac{40 \times 5^3}{12} = 416.67 \text{ in.}^4$$

$$I_v/I_h = 1.95$$

$$E = 1000 f_c' = 1000 \times 4000 = 4 \times 10^6 \text{ psi}$$

$$L_h = 11 \text{ in.}$$

$$L_v = 7 \text{ in.}$$

ANALYSIS BY METHOD I

As only one material is used, the modulus of elasticity can be eliminated from the calculations, so we shall use the following relative values for the moment of inertia: $I_h = 1$ and $I_v = 1.95$.

Computation of isostatic moments

For computing the isostatic moments we use Newmark's numerical procedure. Since the beam is symmetrical it is only necessary to obtain the values for one half of it. Note that the results obtained are expressed as a function of L, the constant distance between risers.

Computation of end moment

After obtaining the isostatic moments, the areas A_{Mh} and A_{Mv} are found from the bending moment diagram. These values divided by the related moments of inertia will furnish the loads on the analogous column θ_h and θ_v.

The general formula for obtaining the moments for any point in the structure is

$$M = M_i - \frac{\theta}{A} + \frac{\theta_h - \theta_v \frac{I_{xy}}{I_x}}{I_y - \frac{I_{xy}}{I_x} I_{xy}} x + \frac{\theta_v - \theta_h \frac{I_{xy}}{I_y}}{I_x - \frac{I_{xy}}{I_y} I_{xy}} y$$

TABLE 3 — FIXED END MOMENTS FOR CASE OF TWO TREADS AND ONE RISER

Equilibrium equations	Moment equations
$M_{BA} + M_{BB'} = 0$ $\frac{M_{AB} + M_{BA}}{11} + 290 = 0$	$M_{AB} = 155.15 \times 10^6(\theta_B - 0.272727 d_B)$ $M_{BA} = 155.15 \times 10^6(2\theta_B - 0.272727 d_B)$ $M_{BB'} = 476.19 \times 10^6(2\theta_B + \theta_{B'})$
Symmetry equation	
$\theta_B + \theta_{B'} = 0$	

where

M_i = isostatic moment of the point under consideration
θ = total load on the analogous column
a = vertical distance from the center of application of the load to the centroid of the analogous column
b = horizontal distance from the center of application of the load to the centroid of the analogous column
I_x = moment of inertia of the analogous column with respect to the x-axis
I_y = moment of inertia of the analogous column with respect to the y-axis
I_{xy} = product of inertia
A = area of the analogous column
x = abscissa of the point under consideration
y = ordinate of the point under consideration
M = bending moment of the structure at the point with coordinates x and y

As the resultant of the loads is applied in the centroid of the section of the analogous column, the formula reduces to

$$M = M_i - \frac{\theta}{A}$$

which gives the value of the bending moment directly for any point of the structure.

Fig. 5 shows the calculation required for the analysis of this example. As can be seen, it not only gives the end moment (—77.8 in-kips) which is the maximum, but also the bending moment at any point in the structure.

ANALYSIS BY METHOD II

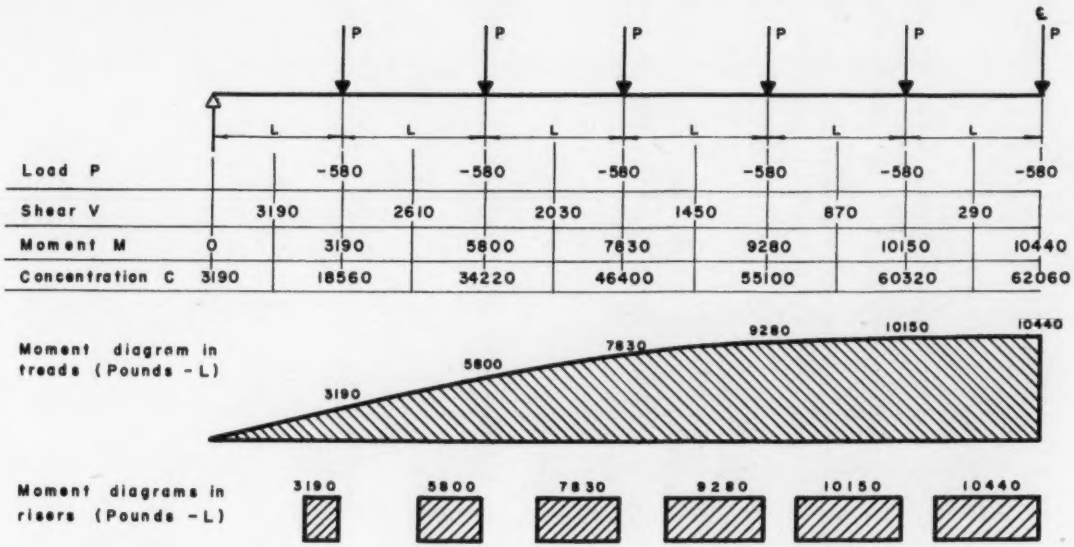
Equation factors

$$K_h = \frac{2EI_h}{L_h} = \frac{2 \times 4 \times 10^6 \times 213.33}{11} = 155.15 \times 10^6 \text{ in.-lb}$$

$$K_v = \frac{2EI_v}{L_v} = \frac{2 \times 4 \times 10^6 \times 416.67}{7} = 476.19 \times 10^6 \text{ in.-lb}$$

$$\frac{3}{L_h} = \frac{3}{11} = 0.272727 \text{ in.}^{-1}$$

$$\frac{3}{L_v} = \frac{3}{7} = 0.428571 \text{ in.}^{-1}$$



$$a_{Mh} = \frac{L^2}{6} \Sigma C = \frac{11^2}{6}(2 \times 248{,}820) = 10{,}035{,}740$$

$$a_{Mv} = V \times L \Sigma M = 7 \times 11\,(2 \times 41{,}470) = 6{,}386{,}380$$

Analogous area

$$A_h = 12 \times 1 \times 11 = 132$$

$$A_v = 11 \times \frac{7}{1.95} = \frac{39}{A = 171}$$

$$M = M_i - \frac{a}{A}$$

$$M = M_i - \frac{13{,}310{,}807}{171}$$

$$M = M_i - 77841$$

Analogous Load

$$\theta_h = \frac{10{,}035{,}740}{1} = 10{,}035{,}740$$

$$\theta_v = \frac{6{,}386{,}380}{1.95} = \frac{3{,}275{,}067}{\theta = 13{,}310{,}807}$$

for the end: $M_i = 0$

$M = -77.8$ kips - in

Fig. 5—Computation summary using column analogy method

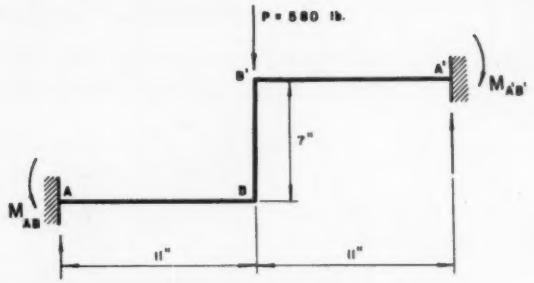


Fig. 6—Case of two treads and one riser

Calculation of differences and of the fixed end moment

For the case of two treads and one riser (Fig. 6) the moment equations are shown in Table 3. Substituting M_{AB}, M_{BA}, and $M_{BB'}$, as functions of rotations and displacements in the equations of equilibrium, the magnitudes of θ_B, $\theta_{B'}$, and d_B can be determined, and using them in the moment equations permits us to obtain their values. The results are

$$\theta_B = -\theta_{B'} = 2.88028 \times 10^{-6} \text{ radians; } d_B = 53.53631 \times 10^{-6} \text{ in.; } M_{AB} = -1818 \text{ in.-lb, } M_{BA} = -M_{BB'} = -1372 \text{ in.-lb}$$

The diagrams of deformation D, of shears V, and of bending moments M for this case are shown in Fig. 7.

Similarly, in the case of three treads and two risers, and using identical substitutions, as shown previously, the results are

$$\theta_B = 18.59922 \times 10^{-6} \text{ radians; } \theta_C = -\theta_{C'} = 11.26112 \times 10^{-6} \text{ radians; } d_{AB} = 177.68530 \times 10^{-6} \text{ in., } d_{BC} = 104.51133 \times 10^{-6} \text{ in.; } M_{AB} = -4633 \text{ in.-lb, } M_{BA} = -M_{BC} = M_{CB} = -M_{CC'} = -1747 \text{ in.-lb}$$

Similarly, in the case of four treads and three risers.

$$M_{AB} = -8500 \text{ in.-lb, } M_{BA} = -M_{BC} = M_{CB} = -M_{CD} = -1073 \text{ in.-lb, } M_{DC} = -M_{DD'} = -4264 \text{ in.-lb.}$$

Note that the maximum value of the moment, following logically, corresponds to the fixed ends of the stairway which is the only one that has to be computed to solve the entire problem. With data obtained from the cases considered, the differences can be calculated as shown in Table 4.

The fixed end moment which will determine the thickness of the tread for 12 effective treads is obtained from Eq. (9)

$$M_{11} = 1.8 + 28.2 + 47.3 = 77.3 \text{ in.-kips}$$

This is practically the same value obtained by Method I.

TABLE 4 — DIFFERENCES RESULTS FOR GENERAL CASE

Span	M	ΔM	$\Delta^{11}M$
2L	1818		
		2815	
3L	4633		1052
		3867	
4L	8500		

STAIRWAY DESIGN

In a particular case of a stairway built and now in service, materials with the following characteristics were used:

Concrete: $f_c' = 4000$ psi; $f_c = 0.45 f_c'$; $f_c = 1800$ psi
Reinforcement: $f_s' = 40{,}000$ psi; $f_s = 0.5 f_s'$; $f_s = 20{,}000$ psi

For $n = 8$ the following factors were obtained: $k = 0.4$, $j = 0.87$, $R = 326$.

The effective depth of the concrete section was

$$d = \sqrt{\frac{77{,}300}{326 \times 40}} = 2.44 \text{ in.}$$

Employing a net covering of 0.5 in. and #3 bars, the total depth of the tread is $2.44 + 0.50 + 0.19 = 3.13$ in., which is less than the value assumed. Taking 4 in. as the total depth the effective depth is 3.31 in. The number of #3 bars required is

$$\frac{77{,}300}{20{,}000 \times 0.87 \times 3.31 \times 0.11} = 13$$

The bars are spaced 3 in. along the width of the stairway.

Bond demands an additional length of bar equal to

$$L = \frac{20{,}000 \times 0.375}{4 \times 0.07 \times 4{,}000} = 6.7 \text{ in.}$$

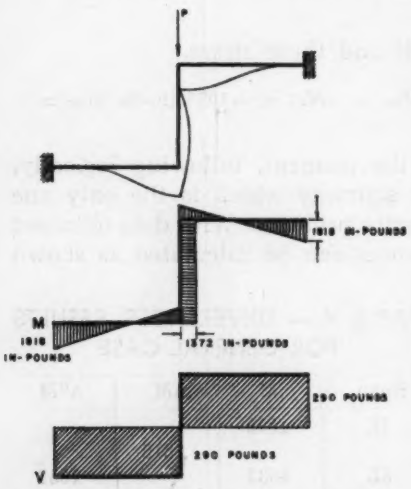


Fig. 7—Deformation, shears, and bending moment diagrams for Fig. 5

This requires the bars to be anchored. Maximum shear creates a maximum diagonal tension equal to

$$\frac{580 \times 11/2}{40 \times 0.87 \times 3.31} = 28 \text{ psi}$$

This amount can be tolerated well by the concrete.

The bending moments vary in magnitude along the length of the stairway but in practice it was decided that it was more economical to reinforce all the steps in the same way, forming rectangular frames which were joined to each other by other bars (also #3 bars), and which also served as temperature reinforcement (see Fig. 8).

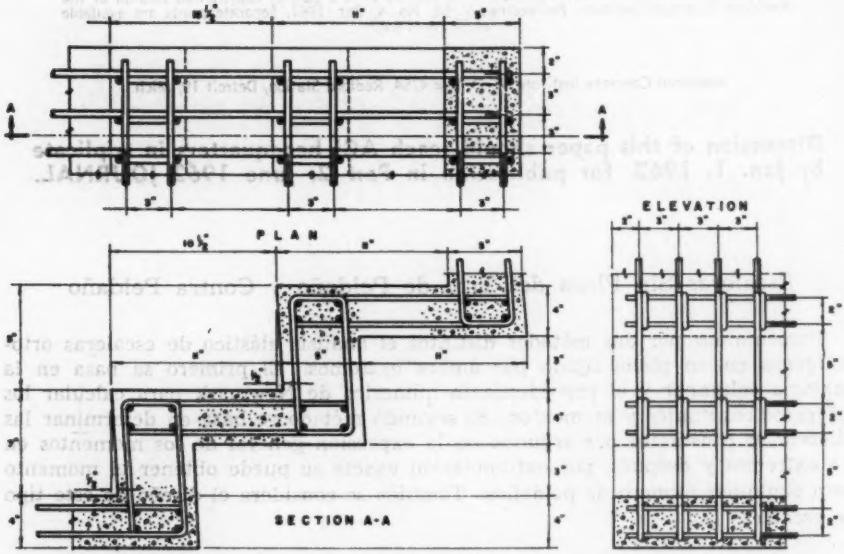


Fig. 8—Plan, elevation, and section of reinforced tread-riser stairway

CONCLUSIONS

1. The analysis of the slabless tread-riser stairs type has been developed independently by two different analytical procedures to check the results. It is proved that both methods give the same results. The fixed-end moments of the example differ by only 0.6 percent which is due to the approximations in the computations.

2. As is usual, both methods employed have disregarded the effect of the axial loads on the elements of the structure.

3. Without a doubt, some other methods may be developed to solve this particular structural problem, and probably may be simpler than the methods used by the authors.

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Escaleras Sin Placa del Tipo de Peldaño y Contra Peldaño

Presentamos por dos métodos distintos el analisis elástico de escaleras ortopolígenas en un plano fijado por ambos extremos. El primero se basa en la analogía columnar y el procedimiento númerico de Newmark para calcular los esfuerzos constantes y momentos. El segundo métado se basa en determinar las diferencias constantes por segundo de la expresión general de los momentos en los extremos y después, por extrapolación exacta se puede obtener el momento para cualquier número de peldaños. También se considera el diseño de este tipo de escalera.

Escalier de Lève-Marche Sans Dalle

L'analyse élastique des cages d'escalier orthopolygonales dans une surface plane attachées dans une et autre extrémitées, est présentée par deux différentes méthodes. La première méthode est basée sur l'analogie de la colonne et la procédures numérique de Newmark pour calculer les efforts tranchant et les moments. La seconde méthode est basée sur la détermination des différences continuelles de secondes de l'expression générale des moments de l'extremité et alors par une extrapolation précise, le moment pour n'importe quel numéro de degrés peut être obtenu. Le dessin de ce type cage d'escalier est aussi débattu.

Plattenlose Stufentreppen

Die elastische Analyse rechtflächiger, an beiden Enden befestigter Treppen wird durch zwei verschiedene Methoden dargestellt. Die erste Methode gründet sich auf die Säulenanalogie und Newmark's numerischem Verfahren zum Berechnen von Scherkräften und Momenten. Die zweite Methode gründet sich auf die Bestimmung der konstanten zweiten Differenzen der allgemeinen Formel für die Endmomente. Durch exaktes Extrapolieren kann dann das Moment für jede Anzahl Stufen erhalten werden. Der Entwurf für diese Art Treppe wird ebenfalls diskutiert.

Title No. 58-18

Strength of Concrete Under Combined Stress

By C. J. BELLAMY

The effect of the intermediate principal stress on the strength of mortar was established for the particular case where the smallest principal stress was zero, by testing hollow cylinders to failure under axial load and confining pressure. The results are compared with some existing failure criteria, and the graphical system of presentation of the criteria examined critically.

■ CRITERIA FOR FAILURE of concrete under the action of combined stresses are of considerable importance in the design of concrete structures, particularly where new and untried forms of construction are proposed. One relatively new application for concrete is in the biological shielding of atomic reactors where the structural design is concerned with the problem of cracking caused by thermal stresses. Such cracking allows radiation streaming and once this occurs the shielding will cease to be effective. A simple way of preventing cracking is to use prestressed concrete; this, however, has the disadvantage of increasing the compressive stresses which may lead to failure elsewhere. For example, in the case of a shield in the form of a hollow cylinder, longitudinal and circumferential prestressing will prevent cracking while the reactor is operating. However, the prestressing can cause a state of biaxial compression at the inside face which is likely to be critical. The ultimate strength of concrete under this condition is important because it influences the amount of prestress that can be applied to the shield and consequently the temperatures and the amount of radiation that it can withstand.

The work described in this report forms part of an investigation into some of the problems involved in shield design. It was undertaken primarily to obtain data on the failure of hollow concrete cylinders under the action of axial load and external pressure, and secondly to determine whether existing failure criteria apply to the condition of biaxial compressive stress so obtained.

EXPERIMENTAL INVESTIGATION

The investigation was planned to reproduce the critical stress condition of biaxial compression as it occurs in the full-sized structure. By applying an axial load and an external pressure to a hollow cylinder it is possible to investigate failure under biaxial compression and for this purpose a triaxial testing apparatus was built to accommodate specimens up to 6 in. in diameter and about 12 in. long. The apparatus was designed so that it could readily be adapted to the testing of solid or hollow cylinders, access being provided to the center of hollow cylinders for electrical wiring or for applying internal pressure.

Apparatus

The essential features of the apparatus are shown in Fig. 1 and 2. The concrete test specimen, sheathed in a neoprene tube (A), is mounted between steel end blocks (B), the seal against the hydraulic fluid between the neoprene tube and the end blocks being made by jubilee clips (C). Hydraulic oil is prevented from getting into the center of the hollow specimens (I) by O-ring seals (D) between the end blocks and their seats. If access to the center of a hollow specimen is not required solid end blocks can be used, thus eliminating the need for the O-ring seals. The specimen is suspended from the piston with flexible wire (E) so that the whole assembly may be lowered into the pressure vessel where an O-ring (F) set into the bore prevents any leakage of hydraulic fluid past the piston. The need for a spherical seat to ensure uniform axial stress was avoided by grinding the ends of the specimen parallel with the aid of a jig. This had no influence on the test results as the comparatively large axial deformations masked any minor irregularities (of the order of $\pm$ 0.003 in.) in the ends of the specimens.

A compression testing machine with ranges up to 1,000,000 lb was used to apply axial loads, and for the tests the platens were set parallel so that the piston was kept squarely in the bore of the pressure vessel. The confining pressure was applied with a small hand pump using light hydraulic oil, and for safety there was a blowout system which released at the maximum allowable confining pressure of 10,000 psi. The confining pressure was measured with gages which were calibrated against a standard dead weight calibrating device. The movement of the piston relative to the pressure vessel was measured by two dial gages mounted diametrically opposite on the top of the vessel as shown in Fig. 1. Tests on a steel specimen 6 in. in diameter and 12 in. long and of known elastic properties enabled a set of curves to be drawn which gave the correction for deflection of the testing apparatus for a wide range of loads and confining pressures.

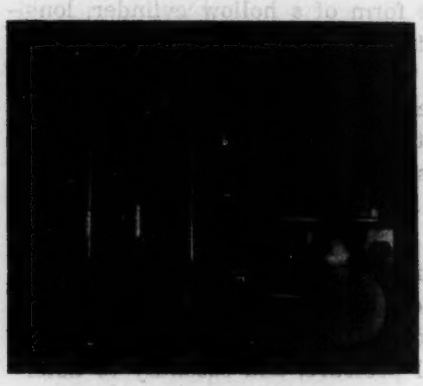

Fig. 1 — Triaxial cell in the Amsler testing machine

The chief sources of error in determining the axial load on the specimen arise from errors in the pressure measurement and from the friction between

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the piston and the pressure vessel. To assess the amount of friction, the piston was loaded against hydraulic pressure alone, and the pressure and load readings taken. By subtracting the total force exerted on the piston by the hydraulic pressure from the load applied, it was found that the friction force was less than 2 percent of the load applied for loads greater than 100,000 lb. Pressures were measured to within ± 20 psi or better, and after allowing for errors in pressure and the friction loss, it is unlikely that the axial load on the specimen at failure is in error by more than 2.5 percent at lower confining pressures. At higher pressures and loads the errors are substantially reduced.

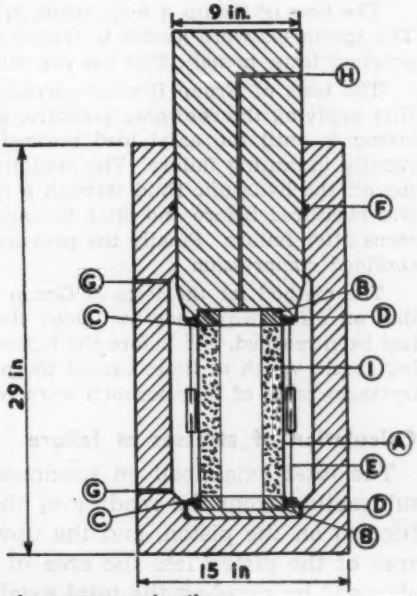
Test specimens

All specimens used were cylinders 6 in. in diameter and approximately 12 in. long, hollow cylinders having a concentric hole with an internal diameter of 2.95 in. They were made of a sand-cement mortar and cast in machined steel molds giving a plane surface on the base, and negligible eccentricity of the central hole. Before testing, the unfinished end of each cylinder was ground parallel to the cast base in a grinding jig.

The mix used throughout was made of a Sydney quartz sand screened to remove foreign particles, and ordinary portland cement with the following proportions:

Sand	2.46 lb
Cement	1.00 lb
Water	0.55 lb at 63 F

This gave a mix which could be conveniently cast into the molds on a table vibrating at 50 cycles per sec. Each batch cast consisted of two hollow and four solid cylinders. The steel molds were stripped after 24 hr, and the specimens transferred to the fog room (100 percent relative humidity and 70 F) where they remained for 7 days after which they were stored at an average humidity of 80 percent and 60 F. This method of curing was chosen so that there would be a minimum rate of increase of strength with time at the age when tested, as it was not convenient to test a complete batch at the one time.



A. neoprene sheath
B. steel end blocks
C. jubilee clips
D. O-ring seals
E. flexible suspension
F. O-ring seal
G. pressure connections
H. access hole
I. hollow concrete cylinder

Fig. 2 — Triaxial testing cell

As it happened the curing time was about 200 days and most batches were tested within a few days, so that variations caused by differences in the curing time were negligible.

Program of tests

The three groups of tests made were as set out below:

Group A — These were intended to show whether there was any significant variation in the compressive strengths given by hollow and solid cylinders. Two hollow and two solid cylinders from each of three batches were tested in unconfined compression.

Group B — Mohr's failure enevelope was found by testing under triaxial compression the six remaining specimens from the batches used for Group A.

Group C — Failure under biaxial compression was investigated by subjecting hollow cylinders to axial load and confining pressure. The two hollow cylinders from each of four batches were tested in this manner, and two of the solid cylinders were used to establish the unconfined compressive strength.

Testing methods

The tests of Group A were made in a 200,000 lb compression testing machine. The specimens were loaded to failure at the rate used for all unconfined compression tests, namely 2300 psi per min.

The tests of Group B were carried out in the triaxial testing apparatus by first applying the confining pressure, and then the additional axial load by increments until the axial load reached a maximum value and then decreased rapidly indicating failure. The confining pressure was kept constant by bleeding off the hydraulic fluid through a needle valve. The length of the specimen was measured before and after testing, and photographs taken of typical specimens after failure. Finally the previously loaded specimens were tested in unconfined compression.

The method for the tests of Group C was the same as for Group B, except that an effort was made to unload the specimen after the ultimate axial load had been reached, and before the hollow cylinder imploded with an accompanying shock, which at times caused the piston to jam in the O-ring seal. In such instances parts of the specimen were reduced to a powder not unlike cement.

Calculation of stresses at failure

The total axial load on specimens in the triaxial cell was found by subtracting from the reading of the testing machine the allowance for friction on the piston, and the upward thrust on an area equal to the area of the piston less the area of the specimen. The axial stress was obtained by dividing the total axial load by the area of the specimen.

The compressive hoop stress, σ_θ, in a cylinder of an elastic material subjected to an external pressure p is given by the conventional equation

$$\sigma_\theta = p \frac{1 + \left(\frac{a}{r}\right)^2}{1 - \left(\frac{a}{b}\right)^2} \qquad (1)$$

where a and b are the internal and external radii, respectively. It was realized that an elastic stress distribution would not exist as the axial shortening of the cylinder at failure, of the order of 0.1 in., indicated considerable nonelastic deformation in the circumferential direction. The circumferential stress distribution at failure must lies between that given by the completely elastic case in Eq. (1) and that given by the condition of uniform circumferential stress. For the latter condition the circumferential stress is given by

TABLE 1 — STRENGTH OF HOLLOW AND SOLID SPECIMENS (AT 245 DAYS)

Batch	Compressive strength, psi	
	Solid	Hollow
1	4900	4480
2	4650	4590
3	4280	4800

$$\sigma_\theta = \frac{pb}{b - a} \quad \text{(2)}$$

Before a failure criterion in terms of stress can be considered it must be known at which point in the cylinder failure is initiated. If the failure is initiated at the inside face it is caused by biaxial compression, but if it is initiated at the outside face it is caused by a state of triaxial compression where all three principal stresses are of a different magnitude. In fact, the first sign of failure in the hollow cylinders was the spalling of pieces of mortar from the inside face, which indicates that failure started there. It is conceivable, but unlikely, that the failure started at the outside face where the circumferential stress could be lower, but it must be remembered that there the third principal stress is not zero and the material would take higher stresses under these conditions. For these tests the circumferential stress at failure lies between the values given by Eq. (1) and (2), i.e., $1.97\ p \leq \sigma_\theta \leq 2.64p$. By using Eq. (1) the results obtained are conservative, that is, the increase in the major principal stress at failure caused by an increase in the intermediate principal stress will be too small.

Test results

Group A — The results of the tests in Group A are shown in Table 1 where the compressive strengths given are the average of two tests.

The mode of failure for both types of specimen was typical of any mortar. A shear cone formed at one or both ends of the specimen and caused longitudinal splitting as shown in Fig. 3(a) and 3(b). Even from the small number of specimens tested (12) it can be seen that for the particular size of cylinder used there is no significant difference between the compressive strength of the solid and hollow cylinders. McHenry and Karni[1] in working with hollow cylinders 24 in. long, with the ratio of wall thickness to diameter equal to one seventh, found up to 20 percent drop in the compressive strength. Here the ratio was one quarter for specimens half as long so that a significant difference would not be expected.

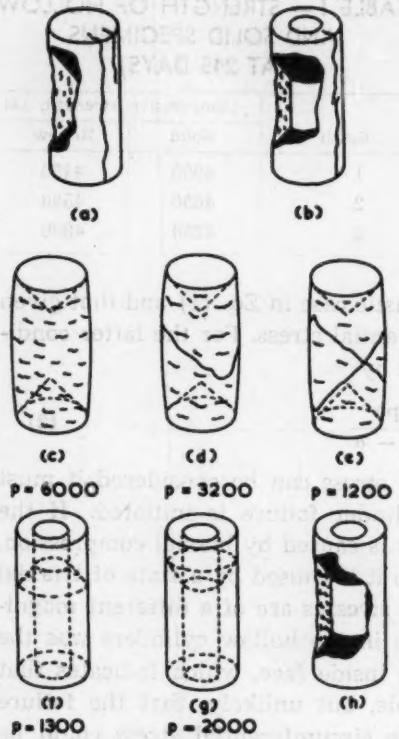


Fig. 3 — Typical types of failure

Group B — Table 2 contains the results of the triaxial compression tests, and shows agreement between results of tests at the same confining pressure.

Modes of failure for the three confining pressures are shown in Fig. 3(a), 3(d), and 3(e), and some photographs of tested specimens in Fig. 4. When the failed specimens were tested in unconfined compression distinct conical surfaces were apparent making an angle of about 60 deg with the horizontal. These were no doubt parts of the original surfaces of failure.

Group C — The results of Group C are contained in Table 3. The tests conducted at the same confining pressures (3200 and 2000 psi) show good agreement, and indicate the ease of reproducing a given set of conditions.

For the hollow cylinders the modes of failure were more valid. Typical cases are represented in Fig. 3(f) and 3(g). In all cases the first sign of failure was the spalling of material from the inside face of the cylinder which occurred at the ultimate load, and the noise that it made could be heard through the access hole in the piston. The mechanism of the spalling was not apparent, though the angles at the edges of larger spalls indicated that they had come from areas bounded by the intersection of shear planes and the surface. Once spalling started the enlargement of the central hole caused increased stresses and hastened failure. Shear cones, with surfaces indicated by the broken lines in Fig. 3(f) and 3(g) were found in the failed specimens though some did not appear until they had been broken. It is interesting to note the change of inclination of the shear planes with increasing confining pressure, i.e., the change from failure with low hoop stress to failure with higher hoop stress. One specimen tested at a confining pressure of 2000 psi showed both types [see Fig. 3(f) and 3(g)] of shear plane failure. At the higher pressures, if the specimen was kept loaded after the ultimate load had been reached, it soon imploded giving the type of failure shown in Fig. 3(h).

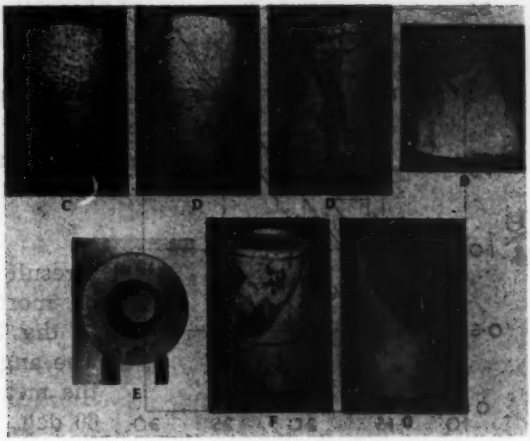


Fig. 4—Typical failed specimens

Analysis of results

Because the tests were conducted on the one material (with constant proportions of sand, cement, and water), the effect of variation of the strength can best be allowed for by expressing all the results in terms of the unconfined compressive strength σ_c. This has been done with the results in Tables 2 and 3, and the modified results are presented in Table 4 which gives the circumferential stress σ_θ, the axial stress σ_z, and the radial stress σ_r in terms of the unconfined compressive strength. The pairs of results in Tables 2 and 3 have been averaged.

From the triaxial compression tests a Mohr failure envelope was drawn (see Fig. 5), and on this two results, the first and last of the

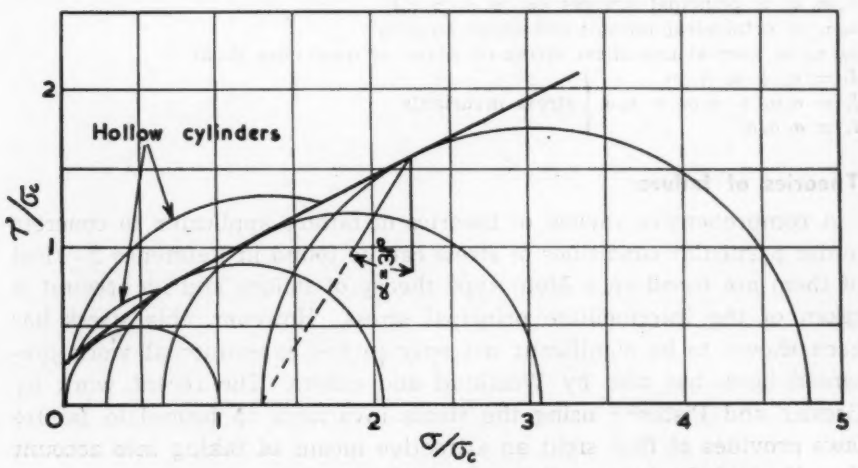


Fig. 5 — Mohr failure envelope for the triaxial test results in Table 4

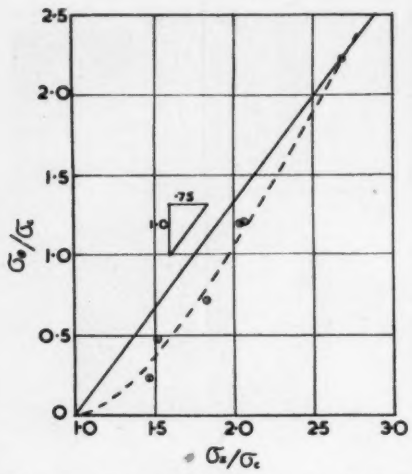


Fig. 6 — Stresses at failure in hollow cylinders

biaxial tests have been plotted to illustrate the magnitude of the effect of the intermediate principal stress. It can also be noted that for the triaxial tests the ultimate axial stress was increased according to the relationship.

$$\sigma_z = \sigma_c + 3p$$

a result agreeing with similar tests on mortar by Smith and Brown at the University of Washington. The angle α (31 deg) agrees with the measured shear cone angle of 60 deg.

The results of the biaxial tests are plotted in Fig. 6 with σ_θ/σ_c versus σ_z/σ_c and the broken line curve shows the implied relationship between the stresses. For the range of stresses shown, Line OA represents the safe criterion that in the particular state of biaxial compression obtained in the tests, the major principal stress at failure is greater than the unconfined compressive strength of 0.75 times the intermediate principal stress.

CURRENT THEORIES OF FAILURE

Notation

$\sigma_1, \sigma_2, \sigma_3$ = principal stresses ($\sigma_1 > \sigma_2 > \sigma_3$)
σ_o, τ_o = octahedral normal and shear stresses
σ_m, τ_m = normal and shear stress on plane of maximum shear
$I_1 = \sigma_1 + \sigma_2 + \sigma_3$
$I_2 = \sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1$ } stress invariants
$I_3 = \sigma_1\sigma_2\sigma_3$

Theories of failure

A comprehensive review of theories of failure applicable to concrete under particular conditions of stress can be found in Reference 2. Most of them are based on a Mohr type theory of failure and no account is taken of the intermediate principal stress. However, this stress has been shown to be significant not only in the experimental work presented here, but also by Wästlund and others. The recent work by Bresler and Pister[2, 3] using the stress invariants to formulate failure laws provides at first sight an attractive means of taking into account the effect of the intermediate principal stress.

TABLE 2 — TRIAXIAL COMPRESSION TESTS ON SOLID CYLINDERS

Unconfined compression strength (p), psi (average of two tests)	Confining pressure, psi	Ultimate axial stress, psi	Unconfined compression strength, psi (after failure)	Permanent axial shortening, in.	Mode of failure (see Fig. 3)
4650	6000	21,600 23,100	1700	1.10	(c)
4900	3200	14,900 15,200	1400	0.85	(d)
4280	1200	8,850 8,850	1700	0.40	(e)

The theory that octahedral shearing stress is a function of the octahedral normal stress at failure, discussed by Bresler and Pister, can be expressed as follows:

$$\tau_o = f(\sigma_o)$$

where

$$\tau_o = F(I_1 I_2) = \tfrac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$

and

$$\sigma_o = \frac{I_1}{3} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$

It is argued that the effect of the third stress invariant I_3 may be neglected, and this is certainly the case when one of the principal stresses is zero, as the invariant I_3 then has a constant value of zero.

Comparison of test results

The test data available on failure under combined stress unfortunately comes from tests on concretes of varied strengths and composition. To compare the results of tests on a concrete of constant composition but with small variations in strength it is not unreasonable to express all stresses in terms of the unconfined compressive strength (σ_c). For concretes made of different materials and of different proportions the comparison could be misleading, but through lack of an alternative this procedure has been adopted here.

The results of the biaxial compression tests in Table 3, together with the results given by Bresler and Pister[3] have been plotted in Fig. 7 in the form τ_o/σ_c versus σ_o/σ_c. Two sets of points representing the upper and lower bounds for σ_θ are shown, and a line parallel to and between these two lines must represent the actual conditions of stress at failure. As the third stress invariant (I_3) is zero for both sets of results, the one curve for τ_o and σ_o at failure should apply. Fig. 7 shows that the straight-line criterion given by Bresler and Pister does not apply beyond the range of their tests.

TABLE 3 — BIAXIAL COMPRESSION TESTS ON HOLLOW CYLINDERS

Unconfined compression strength, psi (average of two tests)	Confining pressure, psi	Hoop stress, σ_θ, psi	Ultimate axial stress, psi	Permanent shortening, in.	Mode failure (see Fig. 3)
4725	430	1140	6,970	0.02	(f)
4725	870	2300	7,200	0.06	(f)
4730	1300	3430	8,650	0.11	(f)
4730	2170	5730	9,750	0.11	(g)
4415	2000	5280	9,000	0.13	(g)
4415	2000	5280	9,100	0.13	(g) and (f)
3765	3200	8450	10,100	*	(g)
3765	3200	8450	10,100	*	(g)

*Specimens did not remain intact.

As the particular interest lay in a failure criterion for compressive stresses, all available test results for failure under combined compressive stresses were plotted on one graph (Fig. 8) with τ_o/σ_c versus σ_o/σ_c. The results plotted include all the tests of Richart, Brandtzaeg and Brown,[4] Balmer,[5] and those given in this report; it will be noted that a smooth curve can be drawn through any particular set. The scatter of points suggests some or all of three things; that the effect of the third stress invariant I_3 is significant, that it is misleading to compare the results of tests on different types of concrete, and that the failure theory is

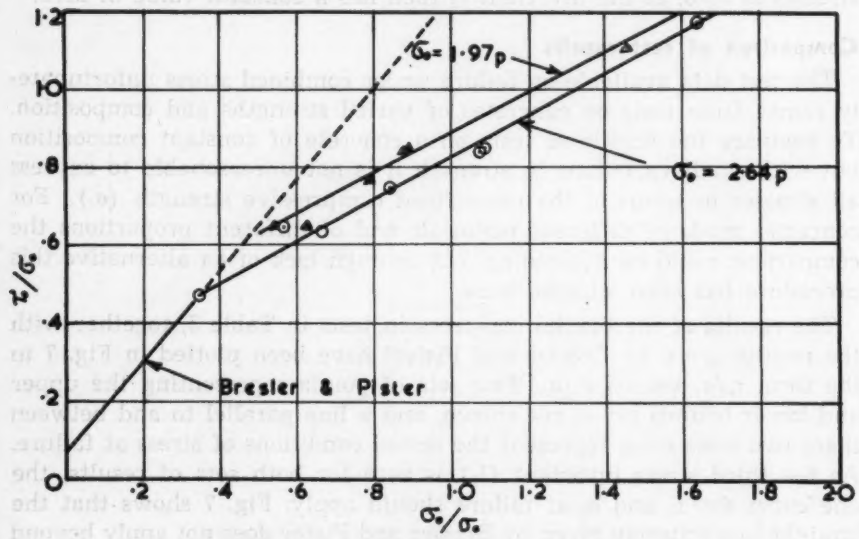


Fig. 7 — Mode of failure curves from Table 3

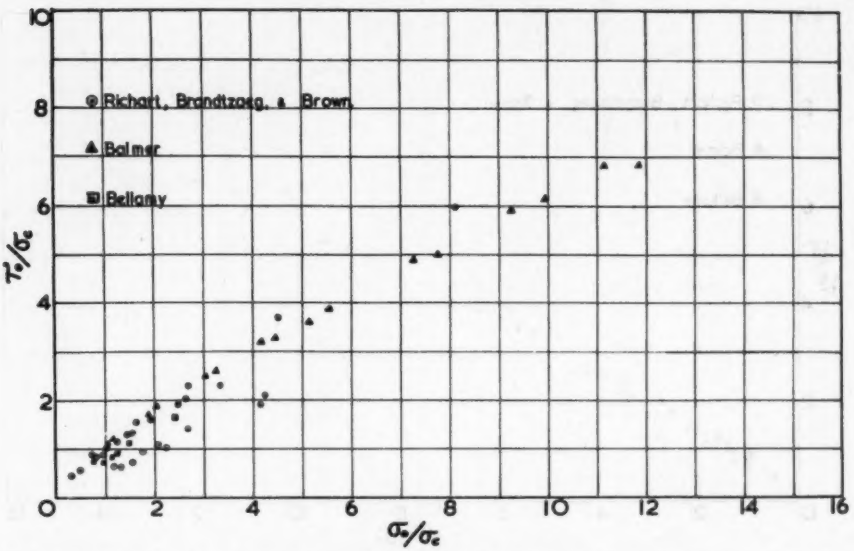


Fig. 8 — Results of compressive stress tests by Richart, Brandtzaeg, and Brown; Balmer; and Bellamy

unsatisfactory. No correlation could be found between the positions of all the points and the values of I_3.

If for the same test results the maximum shear stress is plotted against the normal stress in the form τ_m/σ_c versus σ_m/σ_c (Fig. 9), there is markedly less scatter of the points. Here there may be a correlation between the positions of points and the intermediate principal stress σ_2, but if the effect of σ_2 is as great as the results indicate, the form of presentation does not show it up sufficiently well for any degree of accuracy to be achieved.

Accuracy of graphical presentation using octahedral stresses and maximum shear stress

The use of a failure criterion of the form $\tau_o = f(\sigma_o)$ or of a similar form involving maximum shear stress is open to serious objections, even if the experimental evidence appears to support such a criterion. It was noticed that in some cases large changes in one of the principal stresses did not show as marked deviations from either the τ_o/σ_o or the τ_m/σ_m failure line. This observation has also been made by McHenry and Karni[1] who stated in regard to a failure criterion relating octahedral stresses that "it appears that a surface of failure defined by such a relationship might permit as a close approximation some stress combinations which in terms of principal stresses would be found to be strictly inadmissible."

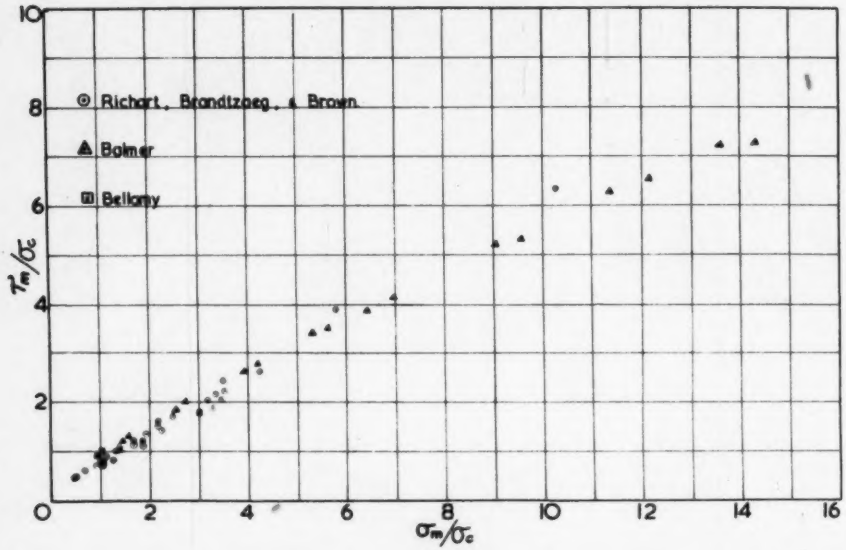


Fig. 9 — Maximum shear stress versus normal stress σ from test results of Richart, Brandtzaeg, and Brown; Balmer; and Bellamy

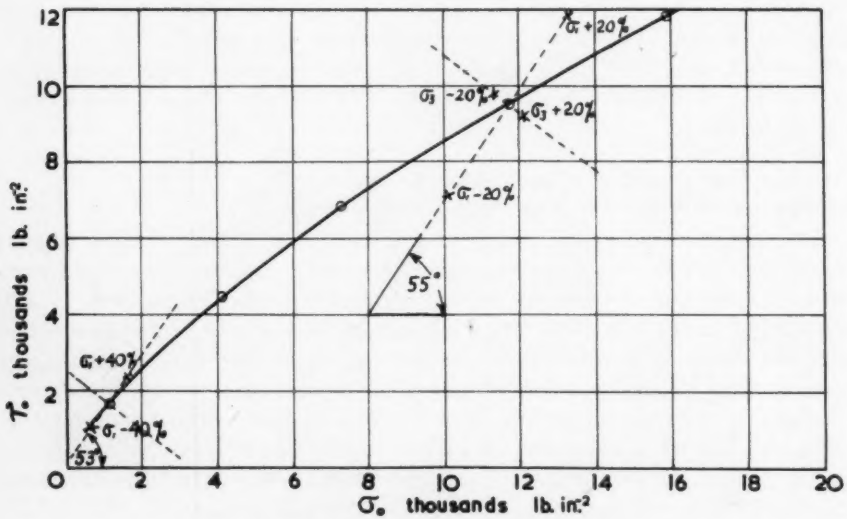


Fig. 10 — Effect of varying σ_1 and σ_2

To investigate this matter, particular examples were selected from Balmer's results[5] and the major or minor principal stress was varied by ± 20 and ± 40 percent. In Fig. 10 is shown the τ_o/σ_o curve for Balmer's results with the almost orthogonal lines obtained by varying σ_1 and σ_3. Lines at about 55 deg to the σ_o-axis were obtained by varying σ_1. Unfortunately, the slope of the failure line given by Bresler and Pister is approximately 50 deg, so that large variations in the major principal stress would pass almost unnoticed in that region of a τ_o/σ_o diagram. Variation of σ_3 while giving a line almost at right angles to the failure curve causes only a small displacement of the point. The plot of τ_m/σ_m also suffers from the same faults, the orthogonal lines obtained by varying σ_1 and σ_3 having a slope of 45 deg.

TABLE 4 — PRINCIPAL STRESSES AT FAILURE IN TERMS OF UNCONFINED COMPRESSIVE STRENGTH

	$\frac{\sigma_z}{\sigma_c}$	$\frac{\sigma_\theta}{\sigma_c}$	$\frac{\sigma_r}{\sigma_c}$
Triaxial tests	4.80	1.29	1.29
	3.07	0.65	0.65
	2.07	0.28	0.28
Biaxial tests	1.48	0.24	0
	1.52	0.49	0
	1.83	0.72	0
	2.06	1.21	0
	2.05	1.20	0
	2.69	2.24	0

Further for the case of compressive principal stresses, the maximum value of the ratio τ_o/σ_o is $\sqrt{2}$ corresponding to a line inclined at 54 deg 40 min to the σ_o-axis. This can be shown analytically and has been confirmed by the calculation of τ_o and σ_o for a large number of sets of three principal stresses selected from random numbers in the range 0 to 10,000. As the slope of the line corresponding to the failure results plotted in Fig. 8 is approximately 40 deg and that given in Reference 2 is nearly 50 deg, the region in which it is possible for a point representing an unsafe stress condition to lie is bounded by two lines including a small angle less than 15 deg.

For these reasons both the octahedral and maximum shear stress theories of failure are unsatisfactory for graphical presentation of failure criteria. A failure criterion based on quantities directly proportional to, rather than derived from the stresses under consideration in a particular system of stresses would not suffer these defects. Accordingly, the most satisfactory approach to any ultimate strength problem is to test the concrete to be used under similar conditions of stress to those expected in practice and to obtain a failure criterion in terms of these stresses.

CONCLUSIONS

By testing hollow concrete cylinders under combinations of axial load and external pressure it is possible to obtain a conservative estimate of the effect of the intermediate principal stress on the failure of con-

crete. For the particular material, and method of testing used in this investigation, the effect of the intermediate principal stress σ_2 is to increase the major principal stress at failure by a minimum of $0.75\sigma_2$.

There is no current theory of failure for concrete with which the test results conform, though it may be possible that they provide an extension for the failure criteria given by Bresler and Pister, for the particular case when the third stress invariant I_3 is zero.

The presentation of failure criteria in terms of τ_o/σ_o, or τ_m/σ_m is unsatisfactory for general graphical use, particularly for compressive stresses less than the unconfined compressive strength of the concrete. Octahedral stresses though providing a neat theoretical means of accounting for the intermediate principal stress, when presented graphically give a false impression because of the inherent inability to distinguish satisfactorily between safe and unsafe states of stress. In general, failure criteria based on quantities directly proportional to stresses will provide the most satisfactory graphical representation.

ACKNOWLEDGMENTS

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Discussion of this paper should reach ACI headquarters in triplicate by Jan. 1, 1962, for publication in Part 2, June 1962 JOURNAL.

Resistencia del Hormigon Bajo Esfuerzos Combinados

El efecto a la resistencia de la mezcla producido por el esfuerzo principal intermediario, se determinó en este caso particular, tomando como base el principal esfuerzo menor partiendo de Cero, utilizándose cilindros huecos hasta que estos fallaron bajo carga axil y presión confinada. Los resultados se comparan con otros criterios de falla, conocidos y el sistema gráfico de la presentación del criterio es críticamente examinado.

Résistance du Béton Sous un Effort Combinée

L'effet de l'effort principale intermédiaire sur la résistance du mortier fut établie pour le cas particulier oú le plus petit effort principal fut un zéro, par l'essai de cylindres creux à la rupture sous une charge axiale et une pression confinée. Les résultats sont comparés avec quelques critères de rupture existants, et le système graphique de la présentation des critéres est examiné avec soin.

Festigkeit von Beton unter Zusammengesetzter Beanspruchung

Die Wirkung de Zwischenhauptkraft auf die Festigkeit von Moertel wurde fuer den besonderen Fall nachgewiesen, wo die kleinste Hauptkraft Null war, indem Hohlzylinder unter axialer Last bis zum Bruch geprueft wurden und man den Druck begrenzte. Die Ergebnisse werden mit einigen bestehenden Bruchkriterien verglichen und das graphische System der Kriteriendarstellung wird kritisch geprueft.

Title No. 58-19

Analysis of Visco-Elastic Behavior of Concrete Structures with Particular Reference to Thermal Stresses

By O. C. ZIENKIEWICZ

The problems of stresses and deformations resulting from imposed loads, displacements, and temperature stresses in a concrete structure are formulated in general terms on the assumption of a visco-elastic behavior of the material. Concrete is assumed to behave as a visco-elastic material with age dependent properties and with a constant Poisson's ratio. Examples illustrating the development of thermal stresses in a simple slab due to the heat of hydration and boundary cooling are presented.

■ TESTS CONDUCTED ON THE creep behavior of concrete over the past three decades indicate that, at least as a first approximation, the stress-strain relationships can be described by the linear theory of visco-elasticity. This means that these relations could be approximated to by a physical model consisting of a large, possible infinite, number of springs (elastic elements) and dashpots (viscous elements) arranged in a suitable combination. Mathematically expressed such a relationship between an uniaxial normal stress σ_x and normal compressive strain ε_x would appear in the form

$$\sum_{m=0}^{n} a_m \frac{d^m}{dt^m}(\varepsilon_x) = \sum_{m=0}^{n} b_m \frac{d^m}{dt^m}(\sigma_x) \qquad (1)$$

Concrete is a material in which the development of strength and other mechanical properties is relatively slow with time. As many of the phenomena of interest take place precisely within the period in which fairly rapid changes of such properties occur, it is necessary to assume that the coefficients a_m and b_m are themselves functions of the time t.

$$a_m = a_m(t)$$
$$b_m = b_m(t)$$

This, in the model, would imply that the stiffness of individual springs and the resistance of the viscous dashpots varies in a predetermined manner with time. The actual determination of the appropriate model is a complex matter and in fact is not necessary for the treatment that follows. Let $c(\tau, t)$ be the solution for σ_x of Eq. (1) for a unit stress applied suddenly at time $t = \tau$ and maintained constant thereafter.

Hence, for an increment of stress $d\sigma_s$ applied similarly at time τ we have

$$d(\varepsilon_s) = c(\tau, t)\, d\sigma_s \quad \text{.....(2)}$$

Since Eq. (1) is linear, and therefore the principle of superposition holds, it follows that the total strain $\varepsilon_s(t)$ at some time (t) due to an application of a time variable stress $\sigma_s(t)$ can be represented by a summation

$$\varepsilon_s(t) = \int_0^t c(\tau, t)\, \frac{d}{d\tau}\,[\sigma_s(\tau)]\, d\tau \quad \text{.....(3)}$$

which, in fact, is the well-known form of the Duhamel integral. In Eq. (3) the tacit assumption of a stress free condition at zero time is made. It should be noted that $c(\tau, t)$ is the creep function or compliance as measured in various tests[1-3] on concrete.* For convenience of notation let the operator

$$\int_0^t c(\tau, t)\, \frac{d}{dt}\,[\sigma_s(\tau)]\, dt \equiv C \quad \text{.....(4)}$$

Therefore, in an operator form Eq. (3) can be written simply as

$$\varepsilon_s(t) = C\,\sigma_s(t)$$

In a general state of stress and strain in three dimensions, and for an isotropic medium, two operators are required to describe the stress-strain relationships as shown by Lee.[5] These are chosen usually in visco-elastic analysis as the relations between the dilative and deviatoric components of stress and strain but here we shall follow the more conventional practice of the theory of elasticity and write for normal strains

$$\varepsilon_x = C\,\sigma_x - N(\sigma_y + \sigma_z) \quad \text{.....(5)}$$

with similar expressions for other components. N stands for an operator of a type similar to Eq. (4) with the kernel giving the lateral strain due to an application of a unit axial compression. It can be shown that the relation between shear strains and stresses can be established in an identical manner to that used in elasticity and that the shear strain

$$\gamma_{xy} = 2(C + N)\,\tau_{xy} \quad \text{.....(6)}$$

with similar expressions for other components.

To draw the analogy between elastic strains and stresses we may observe that

$$C \to \frac{1}{E}, \quad N \to \frac{\nu}{E}, \quad 2(C + N) \to \frac{2(1 + \nu)}{E} = G$$

It is unfortunate that relatively little work has been as yet reported on the creep of concrete under combined stresses.[4] An inference from

*An extensive bibliography can be found in Reference 4.

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the tests of the U. S. Bureau of Reclamation[1,2] is that Poisson's ratio of concrete remains constant during creep and at a low value of 0.1-0.15. A constant Poisson's ratio implies that the operator N bears a constant ratio to the operator C and that Eq. (5) or (6) can be described in terms of one integral operator and a constant. This, if only approximately true, will in general simplify the stress analysis. Similarily it is, of course, possible to define the expressions for stress in terms of strain. It can be observed for instance that under uniaxial stress we can write

$$\sigma_x(t) = \int_0^t r(\tau, t) \frac{d}{d\tau} [\varepsilon_x(\tau)] \, d\tau \qquad (7a)$$

or simply

$$\sigma_x = R \varepsilon_x \qquad (7b)$$

with R standing for the appropriate operator. In this relation $r(t, \tau)$ is the relaxation function representing the stress at time t due to an application of a unit strain at time τ. This leads immediately to the following interpretation of the operators

$$C^{-1} \equiv R \qquad (8)$$

or in analogy with elastic behavior that R is analogous to E.

In a general analysis of stresses and strains in a structure, it is possible to follow the usual elastic procedure replacing the various elastic constants by the appropriate operators. This elastic-viscoelastic analogy is generally attributed to Alfrey[6] but it is worth observing that it was explicitly stated much earlier by McHenry in his classic paper of 1943.[1]

STRESSES AND DEFLECTIONS OF A STRUCTURE CAUSED BY APPLICATION OF A SERIES OF LOADS

Considering a general type of a "bar type" structure of one material placed on unyielding supports and loaded with a series of forces P_1 (t) to $P_n(t)$ each one of which may be applied in some specified manner with time, it is well known that in the case of elastic behavior the following results will be obtained for stresses or reactions and deformations:

$$\sigma_1(t) = a_{11} P_1(t) + a_{12} P_2(t) + \ldots \qquad (9a)$$

and

$$\delta_1(t) = \frac{1}{E_o}\left[b_{11}P_1(t) + b_{12}P_2(t) + \ldots\right] \quad \text{(9b)}$$

In these equations a_{nm} and b_{nm} are coefficients depending only on the geometry of the structure.

If the material of the structure is visco-elastic, an identical analysis can be performed using the analogous operators, the only change occuring now is in the expressions for the displacements which become

$$\delta_1(t) = C\left[b_{11}P_1(t) + b_{12}P_2(t) + \ldots\right] \quad \text{(10a)}$$

or

$$\delta_1(t) = \frac{1}{E_o}\left[b_{11}E_oCP_1(t) + b_{12}E_oCP_2(t) + \ldots\right] \quad \text{(10b)}$$

One may conclude therefore that the stress distribution remains unchanged by creep, i.e., that at any time this can be calculated by the conventional elastic procedure for the loads actually present at the specified time instant and that the deflections could be obtained by assuming an elastic structure with a constant modulus E_o and subjected at the appropriate points to loads equal to $E_oCP(t)$. Each of such equivalent loads can be evaluated by the explicit application of the operator C. Thus the equivalent load P_n' is

$$P_n' = E_o \int_0^t c(\tau, t)\,\frac{dP_n}{d\tau}\,d\tau \quad \text{(11)}$$

This integration can be carried out analytically only if expressions for c and the manner of application of the load are known analytically. This is not generally the case, as usually the function c is given a graphical form from test results. In such instances application of a numerical integration is required. For a sudden application of a constant load P_m at a time τ_o the equivalent load simply becomes*

$$P_m' = E_o\,c(\tau_o, t)\,P_m$$

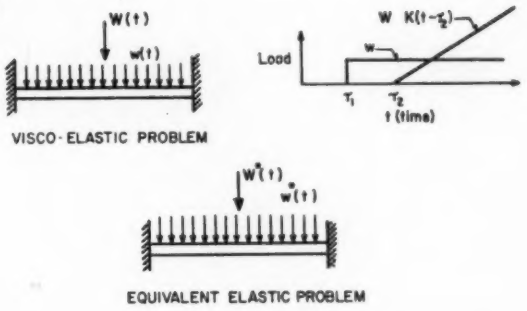
As a simple example of such an integration, consider the deflection of a concrete beam shown in Fig. 1 under the action of two loads: (1) a uniformly distributed load w which is applied at time τ_1 and (2) a concentrated load $W(t)$ which is being increased linearly with time from zero at time τ_2.

$$w(t) = wH(t - \tau_1); \quad W = K(t - \tau_2) \cdot H(t - \tau_2)$$

in which H is the Heaviside step function.

*By the appropriate use of the Heaviside unit step function.

Fig. 1 — Determination of deformations



Central deflections can be obtained for any time t from the equivalent elastic case as

$$\Delta_{max} = \frac{1}{384}\,\frac{L_4}{E_0 I}\,w' + \frac{1}{192}\,\frac{L^2}{E_0 I}\,W'$$

with

$$w' = E_0\,c(\tau_1, t)\,w; \qquad W' = E_0 K \int_{\tau_2}^{t} c(\tau, t)\,d\tau$$

Taking the creep function for example in terms suggested by the investigations outlined in Reference 2 we have for c

$$c = \frac{1}{E_0} + F \log_e [(t - \tau) + 1]$$

in which we shall assume E_o and F are constants. This is approximately true for aged concretes. On integration and substitution we have

$$w' = \{1 + E_0 F \log_e [(t - \tau_1) + 1]\}\,w$$

and

$$W' = \left\{1 + E_0 F \left[\left(1 + \frac{1}{(t - \tau_2)}\right) \log_e (t - \tau_2 + 1) - 1\right]\right\} W(t)$$

This permits the evaluation of displacements at a specific time and it may be of interest to observe that the relative proportion of displacements contributed by individual loads varies with time.

STRESSES AND DEFLECTIONS IN STRUCTURES DUE TO THE IMPOSITION OF SPECIFIED DISPLACEMENTS OR TEMPERATURE CHANGES

If the stresses and deflections of an elastic structure are caused by the imposition of a series of specified displacements, such as support movements, Δ_1, Δ_2, etc., and a series of temperature changes T_1, T_2, etc., then by conventional elastic analysis expressions of a following form

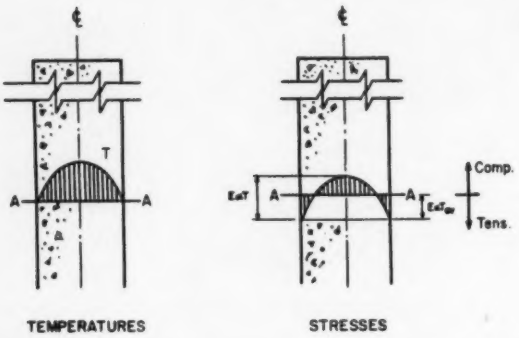


Fig. 2 — Elastic thermal stresses in an unrestrained slab

are obtained for stresses (reactions) and deformations:

$$\sigma_1 = E(K_{11}\Delta_1 + k_{12}\Delta_2 + \ldots + k'_{11}\alpha T_1 + k'_{12}\alpha T_2 + \ldots) \quad \text{(12a)}$$

and

$$\delta_1 = l_{11}\Delta_1 + l_{12}\Delta_2 + \ldots + l'_{11}\alpha T_1 + l'_{12}\alpha T_2 + \ldots \quad \text{(12b)}$$

in which α stands for the coefficient of thermal expansion. Clearly a continuous variation of temperature can be represented by an infinite series of temperature changes at discrete points so the equations are generally valid.

Similarly, we find that if the material is visco-elastic the expression for the displacements remains unchanged but now the expression for stress or reaction becomes

$$\sigma_1 = R(k_{11}\Delta_1 + \ldots) = E_0\left(k_{11}\frac{R}{E_0}\Delta_1 + \ldots \quad k'_{11}\frac{R}{E_0}\alpha T_1 + \ldots\right) \quad \text{(13)}$$

An interpretation of this is simply that while the displacements at any time can be obtained by conventional elastic analysis, the stresses in the structure could be obtained by considering an equivalent elastic structure with a modulus E_o on which displacements and or temperature changes of a magnitude

$$\frac{1}{E_0}R(\Delta)\ ;\ \frac{1}{E_0}R(T) \quad \text{(14)}$$

are applied. Explicitly, these equivalent displacements (or temperature changes) to be applied to obtain the stress picture at a given time t are

$$\frac{1}{E_0}\int_0^t r(\tau, t)\frac{d\Delta}{d\tau}\,d\tau \quad \text{(15a)}$$

and

$$\frac{1}{E_0}\int_0^t r(\tau, t)\frac{dT}{d\tau}\,d\tau \quad \text{(15b)}$$

Thus for example, a sudden sinking of a support of time τ_o, by an amount Δ_o will cause at time t, by Eq. (15a), the same stress as that caused in an elastic structure with a modulus E_o by the application of a corresponding movement of a magnitude*

$$\frac{1}{E_o}\,\Delta_o\, r(\tau_o, t)$$

Since the ratio

$$\frac{r(\tau_o, t)}{r(\tau_o, \tau_o)}$$

gives the relative relaxation, which in concrete at a large time after the application of strain may have a value as low as 1/2 or 1/3, the relation shows that the instantaneous stresses, will, in due course, be reduced in the same proportion.

The application of the concept of "equivalent" temperature changes is of particular importance in problems of thermal stress analysis which arise frequently in mass concrete structures. A simple but illustrative example of such a problem is that of stresses developed in a thick concrete slab due to the heat of hydration of cement. These, as will be observed, are radically changed from stresses which could be predicted by a conventional elastic analysis.

To compute the elastic stresses due to a certain, symmetrical, temperature distribution T, a procedure indicated graphically in Fig. 2 can be followed providing that the slab is unrestrained in the longitudinal direction. This simply states that the stresses at any point are given by subtraction of a uniform, mean, tensile stress $(E\,\alpha\,T)_{avg}$ from the appropriate $E\alpha T$ values. The type of temperature distribution shown in Fig. 2 is typical of the temperatures developed by the heat of hydration and would always, on the basis of elastic analysis, indicate tensile stresses on the faces of the slab. A numerical example will now be considered.

---

*A sudden application of a constant displacement Δ_o at time τ_o is equivalent mathematically to writing

$$\Delta\ (t)\ =\ \Delta_o\ H(t-\tau_o)$$

in which H is the Heaviside step function. Substitution into Eq. (15a) results in the equivalent displacement equal to

$$\frac{1}{E_o}\,\Delta_o \int_0^t r\,\frac{\delta\, H(t-\tau_o)}{\delta\tau}\,d\tau$$

in which

$$\frac{\partial H}{\partial \tau} \equiv \delta$$

and δ is the Dirac impulse function, it can be easily shown to be

$$\frac{1}{E_o}\,\Delta_o\, r\ (\tau_o, t)$$

Careful attention to such detail is at times essential to the correct interpretation of the various formulas given.

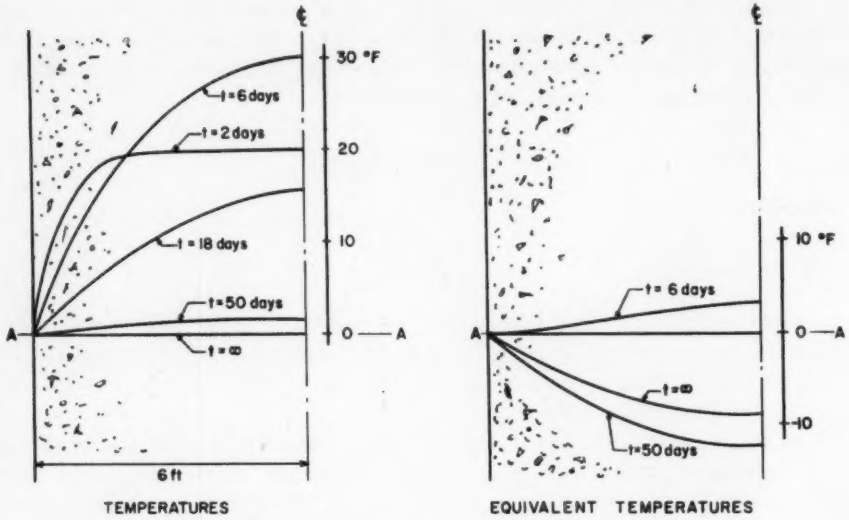


Fig. 3 — Temperatures developed due to heat of hydration.

In Fig. 3 temperatures developed due to heat of hydration in a slab 12 ft thick are shown at various times after casting. These have been computed by the well-known methods of numerical integration assuming that the thermal diffusivity of the concrete is $k = 1$ sq ft per day and that the adiabatic temperature rise (deg F) may be expressed by the following equation:

$$T_{adiabatic} = 70(1 - e^{-0.4t})$$

in which t is the age of the concrete in days.

For simplicity the temperature at the boundaries was assumed constant. This implies a perfect cooling there, never completely achieved in practice.

Now taking into account the modifications introduced by the viscoelastic behavior of the material it is necessary to evaluate the distribution of equivalent temperatures, given by Eq. (15b), at various times. To do this, the knowledge of the relaxation function applicable to the concrete concerned is essential. A form of such a function, which is only approximate, has been suggested by Babuska[7,8] as

$$r(\tau, t) = E_0(1 - e^{-0.07t}) \tfrac{1}{2} (1 + e^{-0.08(t-\tau)})$$

in which E_o is the instantaneous modulus of the concrete at a large age and the origin of time in days is taken at the instant of casting.*

The integration is carried out numerically noting that

$$\frac{1}{E_o}\int_0^t r(t,\tau)\,\frac{dT}{d\tau}\,d\tau \equiv \sum_{n=0}^{n=T/\Delta T} r\,[t,\,(n-\tfrac{1}{2})\,\Delta\tau]\,\Delta T_n$$

in which ΔT_n corresponds to the increment of temperature occurring in the nth interval of τ.

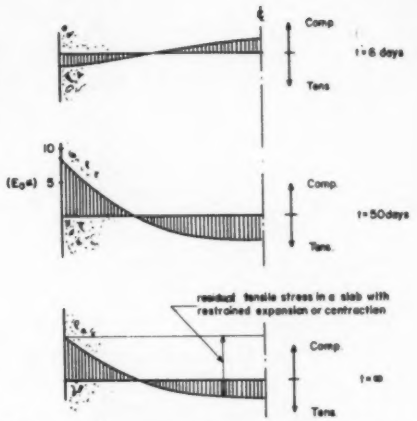


Fig. 4 — Stresses in a slab due to heat of hydration

Such a numerical integration is surprisingly accurate even with fairly large divisions of time. The computed equivalent temperatures for 6 days, 50 days, and a large length of time are shown in Fig. 3 (right). These temperatures applied to an unstrained elastic slab with a modulus E_o (the final modulus of fully hardened concrete) will produce stresses which are equal to those of the visco-elastic problem at appropriate times. Such stresses calculated by the simple principle of Fig. 2 are indicated in Fig. 4. It is an interesting but practically demonstrable fact that a complete reversal of the stress picture predicted by the elastic theory occurs after the concrete has aged. The high interior heat causes only small tensions to occur at the outside of the slab at early ages of the concrete and the subsequent cooling results in an appreciable amount of compressive stresses on the outside faces with low tensile stresses in the center of the block. Such stresses, in fact, remain permanently in the concrete as reduction of tensile cracks.†

A somewhat qualitative description of this effect was presented recently by Holden[9] and attempts to enhance this effect were proposed by Carlson and Thayer.[10] Carlson and Thayer suggest an initial cooling of the concrete surfaces by refrigeration of the forms. The effect of cooling of the forms by 30 F for 12 days was computed in a similar manner and is illustrated in Fig. 5 and 6. Although this occurs simultaneously with the hydration heating process, it can be considered

*This expression is not found to be in good agreement with relaxation functions computed by a student of the author, B. Sridhara, for the data on creep given by the U.S. Bureau of Reclamation.[3] It appears, however, that these cannot be accurate especially for the early ages of the concrete since they result in negative relaxations.

†The process of production of "toughened glass" by rapid cooling is based on similar behavior.

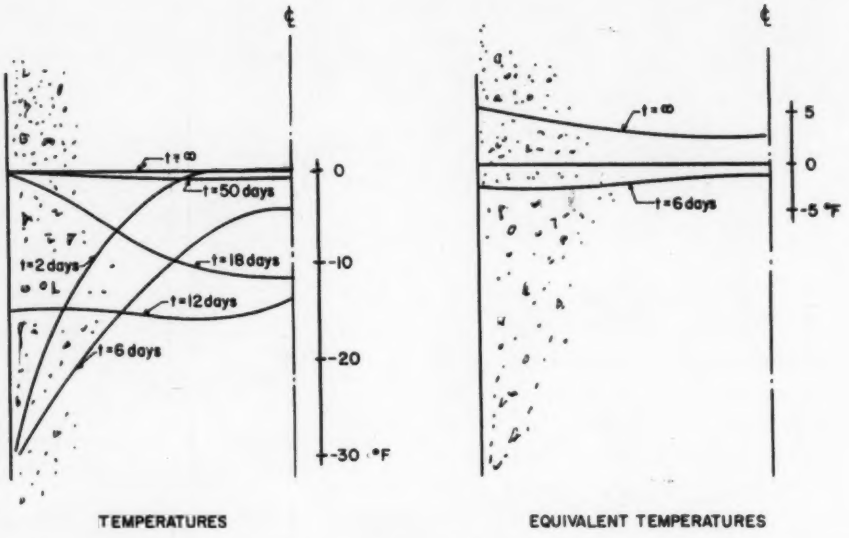


Fig. 5 — Temperatures due to cooling of formwork to 30 F during first 12 days

separately because of the linear nature of the effects. It is interesting to note that the ultimate prestressing achieved is, in this example, somewhat small compared with that due to the heat of hydration. Nevertheless, an indication of the existing possibilities is interesting and may contribute to the development of crack resistant concretes.

Another interesting situation relevant to the same problem is the case in which the slab is restrained against all length changes at all times. In an equivalent elastic case this would amount to compressive stresses of magnitude $E\alpha T$ being developed throughout the section. As, in opposition to the actual temperatures, the equivalent temperatures are negative after a certain time, it is clear that a system of tensile stresses throughout the mass would be "built in." This, in an arch dam for instance, would tend to open contraction joints and justifies the use of grouting.

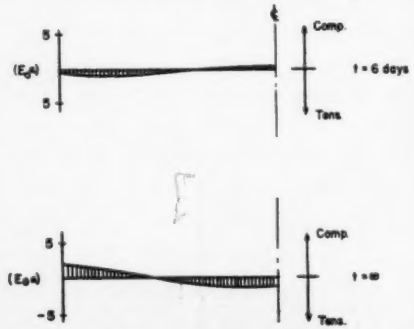


Fig. 6 — Stresses in a slab due to cooling of formwork

CONCLUSIONS

The examples included here are of a simple nature but have, it is hoped, indicated general procedures which are applicable for the evaluation of deformations and stresses in more complex cases. For instance, the problem of thermal stresses in thin slabs in which flexure may occur is amenable to precisely the same method.

It should be noted that a knowledge of creep and relaxation relations for the material is important. These relations are not independent, and in fact one may be derived from the other.[2] The inaccuracies inherent in this derivation make it desirable to develop experimental techniques by which either can be measured independently. To date, it appears that though numerous tests on creep compliance have been undertaken, the direct determination of the relaxation modulus for concrete has not been attempted. It is important that work in this neglected area, as well as experiments on triaxial behavior of concrete be started.

Extension of the approach presented here, to problems of two or three dimensional stress analysis of mass concrete is a matter which presents no more difficulty than the solution of the appropriate elastic problems. Several such problems of various degrees of complexity are at present being investigated by the author using numerical analysis and other techniques.

ACKNOWLEDGMENT

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Discussion of this paper should reach ACI headquarters in triplicate by Jan. 1, 1962, for publication in Part 2, June 1962 JOURNAL.

Analisis del Comportamiento Visco-Elastico o Elasto-Plastico de Estructuras de Hormigon con Particular Referencia a los Esfuerzos Debidos a la Temperatura

Los problemas de esfuerzos y deformaciones debidos a cargas impuestas, desplazamientos y violencias causados por la temperatura en estructuras de hormigón se formulan en sentido genérico considerando que el material se comporta como si fuera de consistencia viscoso-elástico. Se supone que el hormigón se comporta como un material viscoso elástico con propiedades dependientes de su edad y con la razón constante de Poisson. Se dan ejemplos demonstrando como se desarrollan los esfuerzos térmicos en una losa sencilla cuando se presentan la temperatura de hidratación y el enfriamiento en los extremos de la misma.

Analyse de la Conduite Viscose-Élastique des Structures de Béton avec une Particulière Réference aux Efforts Thermiques

Les problèmes des efforts et déformations résultants des charges imposées, les déplacements et les efforts de température, dans une structure de béton, sont formulés en termes généraux sur l'assomption d'une conduite viscose-élastique de la matière. Le béton est supposé de se conduire comme une matière viscose-élastique avec propriétés dépendantes de l'âge et avec une proportion constante de Poisson. Des exemples sont présentés pour illustrer le développement des efforts thermiques dans une dosse simple, dû à la chaleur d' hydration et au refroidissement du bord.

Analyse des viskos-elastischen Verhaltens von Betonbauten unter Besonderer Beruecksichtigung von Waermesponnungen

Die Probleme der Spannungen und Verformungen, die sich aus auferlegten Lasten, Verschiebungen und Waermespannungen in einer Betonstruktur ergeben, werden unter der Annahme eines viskos-elastischen Verhaltens des Materials in allgemeinen Begriffen ausgedrueckt. Es wird angenommen, dass Beton sich wie ein viskos-elastisches Material verhaelt mit vom Alter abhaengigen Eigenschaften und mit einer konstanten Querdehnungsziffer (Poissonsche Konstante). Beispiele werden geboten, welche die Entwicklung von Waermespannungen in einer einfachen Platte erlaeutern, die von der Hydrationswaerme und Randabkuehlung herruehren.

Title No. 58-20

Hyperbolic Reinforced Concrete Cooling Towers

By PAUL ROGERS

This paper describes the general features, operation, and economic advantages of the natural draft type, hyperbolically shaped, reinforced concrete cooling towers.

AMERICANS TRAVELING ABROAD often marvel at the huge reinforced concrete, natural draft cooling towers usually located in the proximity of industrial developments. These structures, sometimes reaching over 350 ft in height and with base diameters often over 200 ft, have a graceful double curvature shape (a hyperboloid), as shown in Fig. 1.

In the United States the engineering profession is more familiar with conventional cooling towers of the forced, or induced draft type. Their purpose is to cool and reuse large quantities of water for industrial use. Immense quantities of water are required for the condensers of power stations, refineries, steel plants, air conditioning, etc.

In general, whether the draft is induced naturally or mechanically, cooling towers operate on the principle that the hot water is pumped to a certain height and then distributed through a piping system to nozzles where it splashes over a system called filling or stacking. Such filling, which may be of wood or transite material, usually consists of closely spaced thin slats placed in several layers. The water splashes, drips, and flows downward through the filling. The air which is forced upward, either mechanically or by atmospheric pressure difference, rises through the driplets or films of the hot

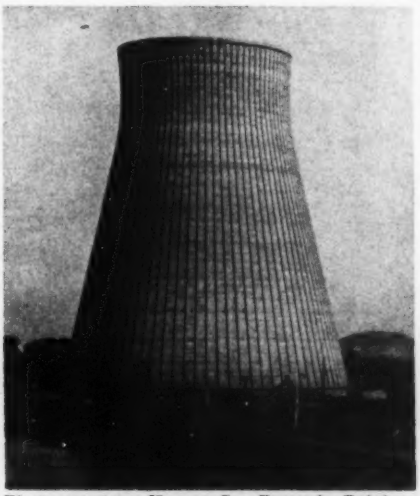


Photo courtesy, Hamon Co., Brussels, Belgium

Fig. 1 — Cooling tower near Brussels, France

water and consequently cools the water through evaporation and convection. The cooled water is collected in a pond at the bottom of the towers and then recirculated for industrial use.

Fig. 2 shows an isometric view of the filling, or cooling stack, composed of thin asbestos sheets placed in several layers and resting on a reinforced concrete column and beam system. The drawing shows the hot water distribution pipes, also of transite material, and the nozzles for splashing the water.

The American cooling tower industry has done an excellent job in developing the conventional type of tower, which is mostly of wood construction. There are, however, definite advantages in building large, reinforced concrete towers of the natural draft type. Some of these advantages are discussed herein.

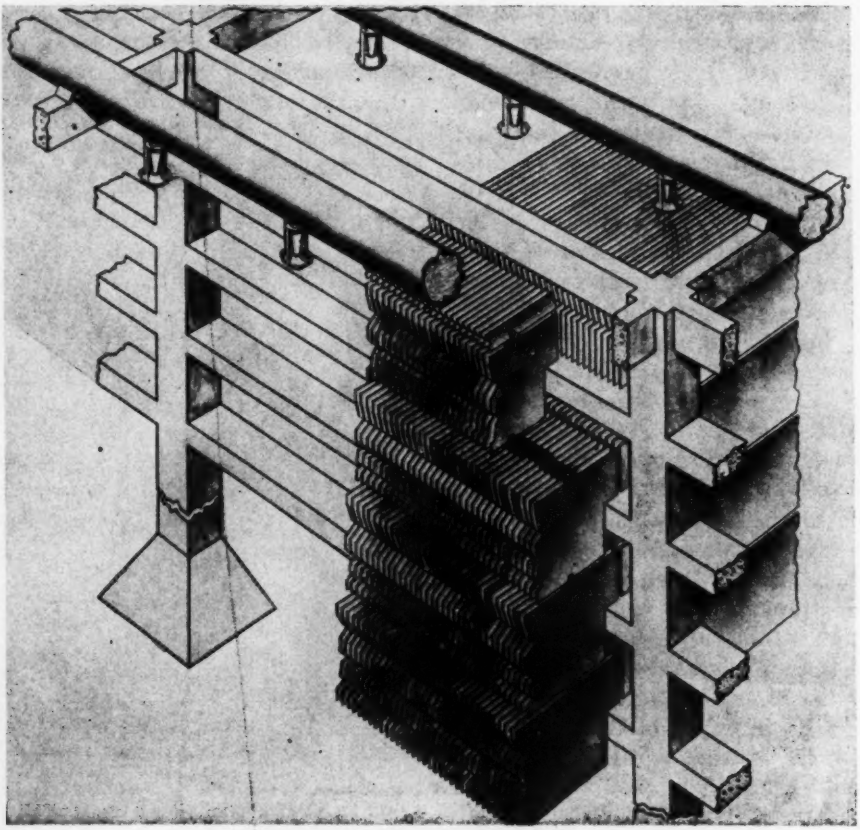


Fig. 2 — Asbestos sheet cooling stacks

ACI member **Paul Rogers** is president of Paul Rogers and Associates, Inc., consulting structural engineers, Chicago, Ill., and is also a partner in Bonnet and Rogers, civil engineers, Paris, France. Mr. Rogers has lectured at universities and is the author of numerous articles including a textbook, "Tables and formulas for fixed-end moments." Since 1953 he has been in private practice and responsible for the design of multistory buildings, industrial plants, and bridges. Active in ACI work, he is chairman of Committee 322, Design of Structural Plain Concrete, and a member of Committee 318, Standard Building Code; Committee 321, Design of Reinforced Concrete Slabs; Committee 336, Combined Footings; and Committee 714, Concrete Bins and Silos.

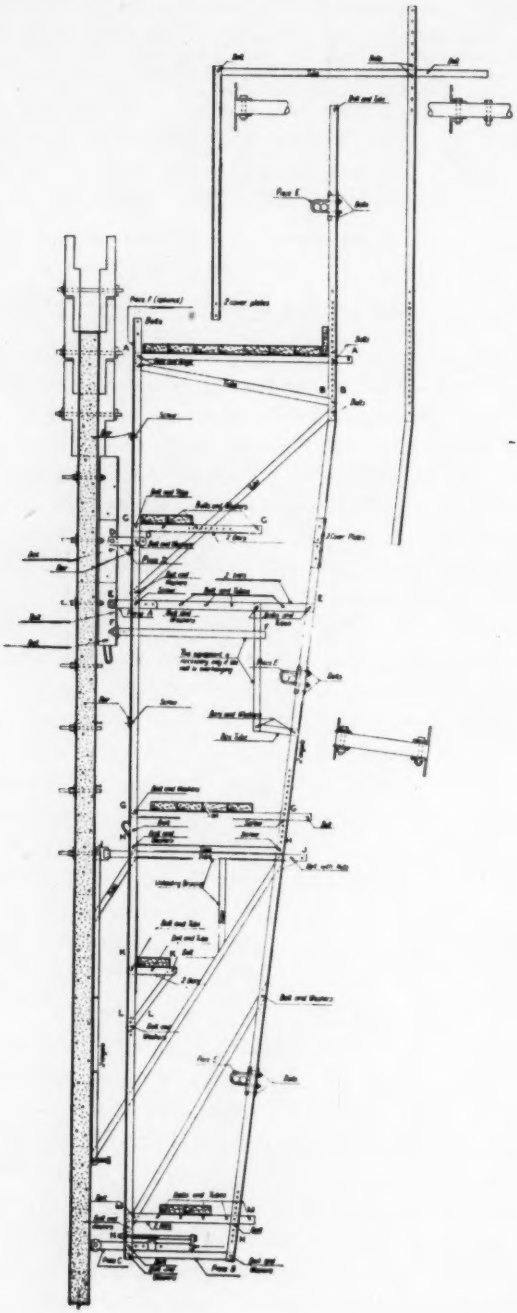
ADVANTAGES OF HYPERBOLIC CONCRETE TOWERS

Concrete towers are permanent. There are cooling towers in Europe which were built more than 40 years ago and are still functioning perfectly with no sign of structural deterioration. Maintenance is nil as far as the thin concrete shell is concerned; if transite filling is used the maintenance for the filling is at a minimum. As there are no fans or similar equipment, there is no need for an extensive electric wiring system, and the only power consumption is needed for pumping the water to the distribution pipes. There are no moving mechanical parts anywhere within these towers.

Advantage of the latent kind is the complete lack of vibration due to resonance of fans and the tower. Fire hazard which plagues many wooden towers during off-season is nonexistent for concrete construction. Tall natural draft towers also minimize such hazards as mist and frozen spray which may represent a danger to equipment and nearby traffic.

Since these towers have an immense moment of inertia there is considerable resistance to high winds, hurricanes, tornadoes, and earthquakes. Due to this and the lack of fire hazards, insurance premiums are usually low. The initial construction cost of natural draft towers is higher than that of conventional cooling towers. However, allowing for a long term amortization and other benefits as stated previously, the ultimate cost is often to the advantage of the owner.

Considering these advantages one would expect a large number of towers throughout the United States. In reality, it was only recently that a contract was let for the first tower in connection with a power station in Kentucky. There are several reasons why the United States is lagging behind in the construction of hyperbolic cooling towers. One of the reasons is that until recently the water supply for power generation and cooling purposes was adequate. Secondly, since the construction of hyperbolic towers calls for advanced mathematical computations and unusual construction practices, there is the tendency to adopt such systems only after long usage abroad.



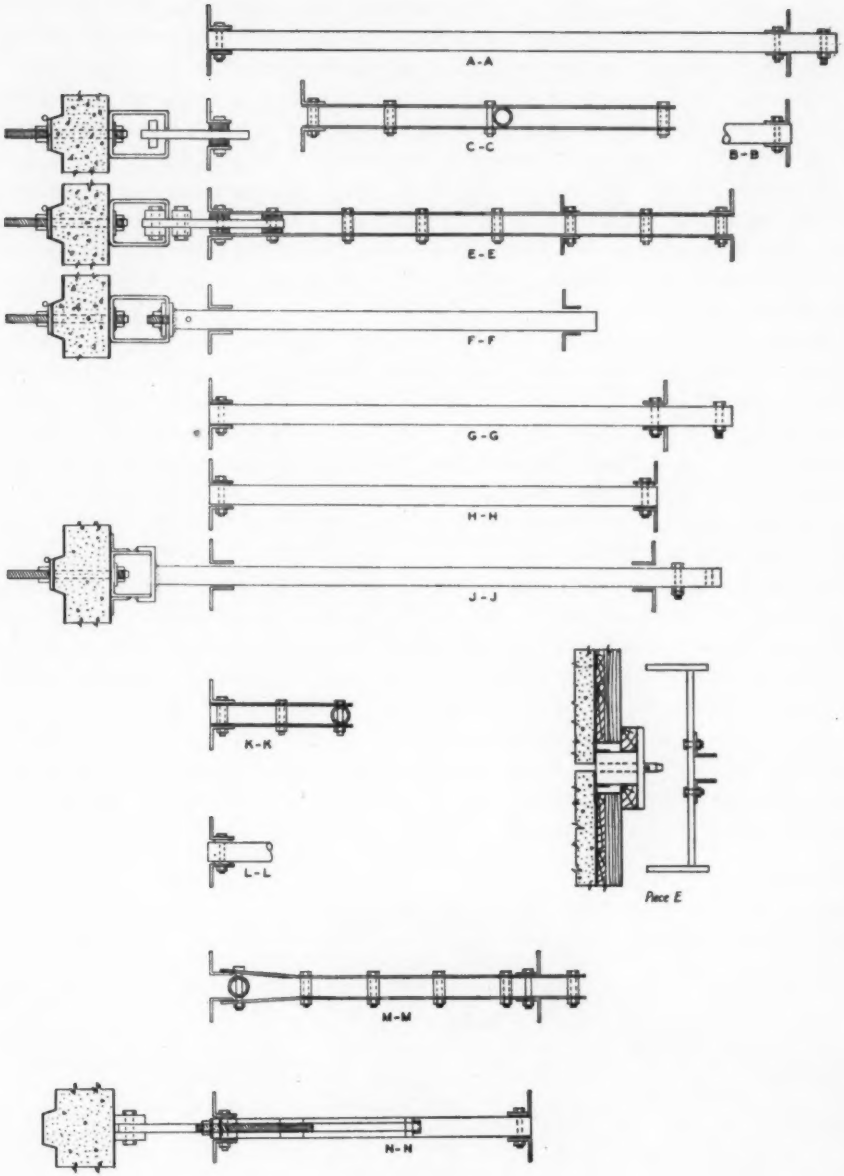


Fig. 3 — Faye method of scaffolding (patented); elevation (left-hand page); typical sections (above)

DESIGN AND CONSTRUCTION

Thin shell construction has become well accepted and adopted in the United States, although most structures have been horizontal structures such as hyperbolic paraboloids and cylindrical shells. The design and construction of a vertical shell structure, such as a hyperbolic cooling tower, requires that designers master the necessary mathematical analyses and that contractors become experienced in this field.

Fundamentally, the hyperbolic shape was adopted because it enables construction of these enormous towers with thin walls (4 to 15 in); because of its shape, the shell is only subject to compression forces induced by normal loading conditions. Thus, such towers could be, and sometimes are, built of precast elements.

When the cast-in-place method is used, the construction of hyperbolic towers calls for considerable ingenuity. Until recently, full height interior scaffolding was employed (mostly of the tubular steel type), but lately the forms for a new "placement" are supported by the previously cast concrete. Contractors will undoubtedly devise different methods of formwork. Nevertheless, they may be interested in the patented Faye system, illustrated in Fig. 3 and 4 which has attained acceptance and success abroad.

It should be noted that the towers built by a Belgian-French firm are vertically fluted externally. Such roughing of the surface results in reduced vortex or eddy formations, and there is less chance for aerodynamic instability. In the same vein, the diagonally cast columns resist possible distortion of the circular cross section.

Fig. 5 shows a half section and a half elevation of a hyperbolic cooling tower. The section shows the stacking or filling, the hot water intake, the water distribution, and the collecting pond from which the cooled water is recirculated.

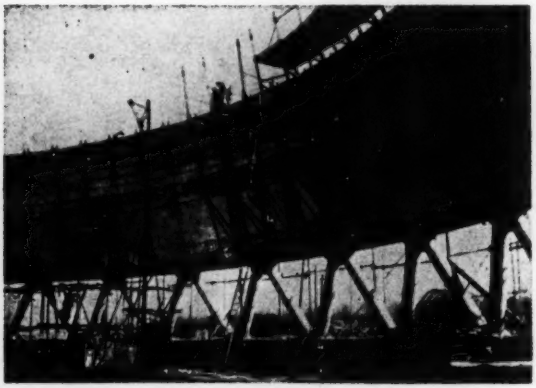


Fig. 4— Faye method of scaffolding in operation

Photo courtesy, Hamon Co., Brussels, Belgium

Only a minimum of theoretical analyses is presented here. Anyone interested in the more advanced theories can study the various papers on shells in the ACI JOURNAL and other excellent publications and books on the subject by Portland Cement Association, Timoshenko, Flügge, Girkman, and others.

The membrane theory assumes that the thickness of the shell is so small compared with its diameter that bending stresses can be omitted. The stress analysis is greatly simplified since the resultant moment and shear forces disappear.

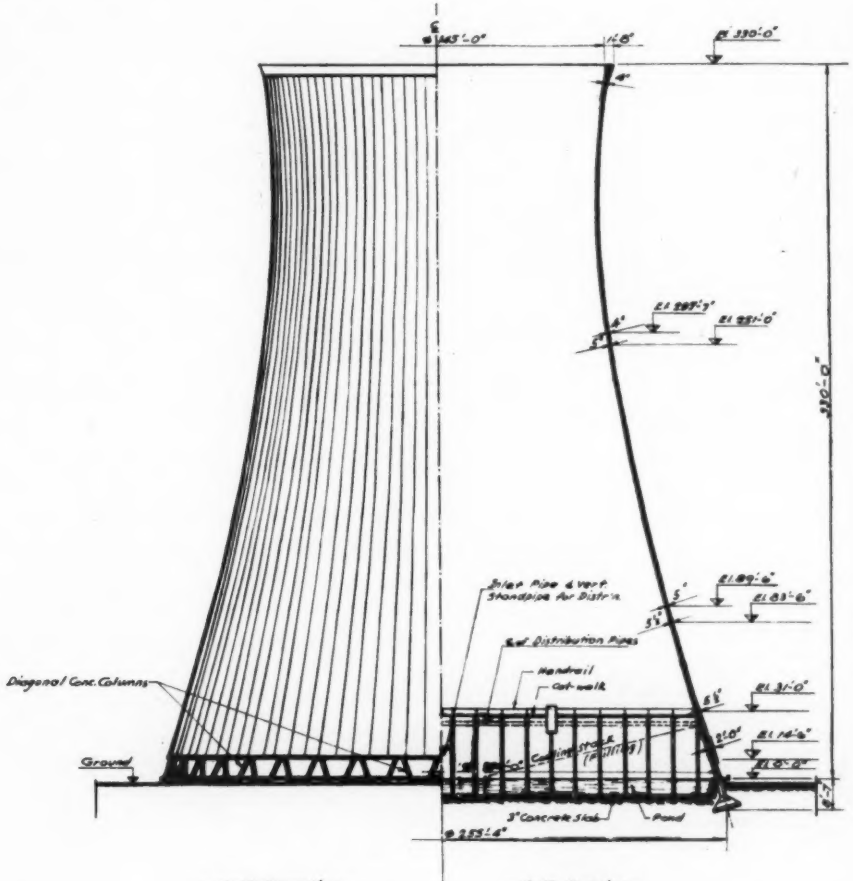


Fig. 5 — Natural draft cooling tower of reinforced concrete

Equilibrium in a direction normal to the wall surface

From Fig. 6 and 7 the area of an element is $r_1\, d\alpha\, r_m\, d\beta$. Then neglecting the dead weight

$$\sigma_m\, t r_1\, d\alpha\, d\beta = \sigma_1\, t r_m\, d\alpha\, d\beta$$

from which

$$\sigma_1 = \frac{r_1\, \sigma_m}{r_m} \qquad (1)$$

The most important stress is due to the dead weight, N_v, of the structure.

$$\sigma_m = \frac{N_v}{2\pi\, r_n \cos^2 \phi t} \qquad (2)$$

If this dead weight is included Eq. (1) becomes

$$\sigma_1 = \sigma_m \frac{r_1}{r_m} + \varrho\, r_1 \sin \phi \qquad (3a)$$

for the part below the neck.

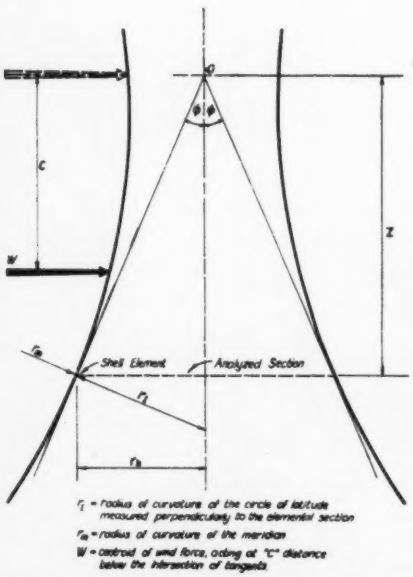

c
W
z
Shell Element
Analyzed Section
r_1
r_n
r_1 = radius of curvature of the circle of latitude measured perpendicularly to the elemental section
r_m = radius of curvature of the meridian
W = centroid of wind force, acting at "C" distance below the intersection of tangents

Fig. 6 — Location of an analyzed shell element

Hence

$$\sigma_1 = \sigma_m \frac{r_1}{r_m} - \varrho\, r_1 \sin \phi \qquad (3b)$$

for the part above the neck.

Wind effect on structure

If W acts at C, the horizontal force $= W(1-c/z)$ and the shear on the element $= W\, c/z$. This term can be added algebraically to Eq. (1), (2), and (3).

From Eq. (1) it follows that σ_1 is also compressive. This is a peculiarity of the hyperboloid and explains why this particular shell shape is so suited for construction in concrete.

The development of differential equations for the solution of internal forces is beyond the scope of this presentation. It is sufficient

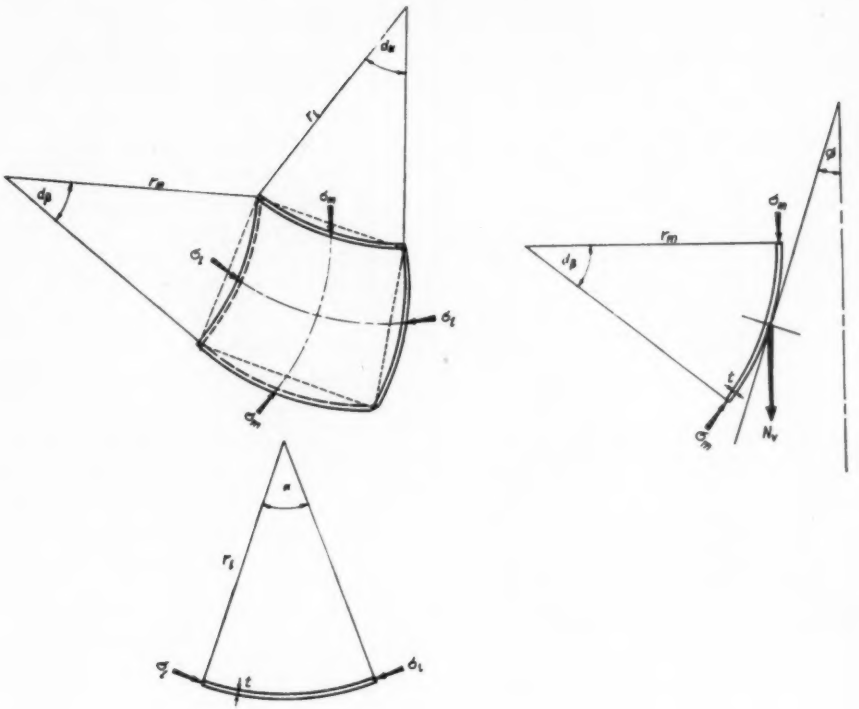


Fig. 7 — Stresses on an elemental surface of hyperbolic shell

to say that only compression stresses exist in the shell under symmetrical circumferential loads; there are neither flexural nor shear stresses present under such conditions.

The analysis of the true hyperbola of revolution is complex and is usually carried out by using a step-by-step method for wind analysis.

The wind forces acting on a structure of this type are distributed as shown in Fig. 8 and 9. This indicates that the shell is subject to positive pressure only on a small area directly in the path of the prevailing wind. The rest of the shell is under negative pressure, or suction. The wind induces both shear and flexural stresses in the shell, for which reinforcement is necessary. Both analysis and practical experience indicate that only nominal reinforcing is needed to resist the minor stresses induced by the wind.

In most cases, the compression forces due to weight dominate to such an extent that there is no reversal of stress. Consequently, a nominal reinforcement, as a rule, is more than adequate for resisting shrinkage and temperature stresses, plus the occasional light flexural stresses.

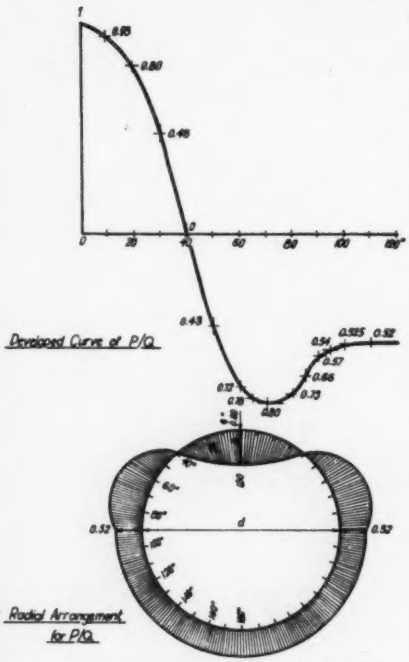


Fig. 8 — Wind pressure distribution

Ring beam

The ring beam connecting the tops of columns, and which forms the lower part of the shell, may be of gradually tapered thickness or with an abrupt transition to the shell. Since the depth to span ratio is in excess of unity, such ring beams should be designed by the deep beam theory.

Column foundation

The column foundations are, as a rule, separated from the footings of the cooling stack supports; the columns carry heavy loads whereas the filling supports are lightly loaded. Furthermore, quite often the shell supporting columns require piling, caissons, or continuous ring foundations, and it is desirable to separate them from the light interior footings.

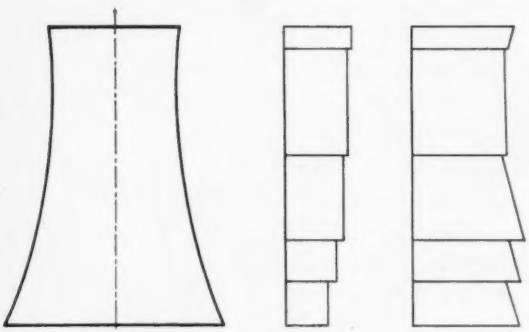


Fig. 9 — Projected area (left); wind pressure per sq ft (center); and wind pressure on projected area (right)

CONCLUDING REMARKS

The author has attempted, within the limits of this paper, to present the design, construction, operation, and advantages of hyperbolic natural draft reinforced concrete cooling towers. These towers, both from the aerodynamic and structural points of view, are superior to any other type of cooling tower facility so far in use. This superiority might well

be the reason that several thousands of such towers have been built in various parts of the world. It is hoped that American engineers will consider this type of tower for their future projects.

Based on a paper presented at the ACI 13th regional meeting, Tucson, Ariz., Nov. 2, 1960. Title No. 58-20 is a part of copyrighted Journal of the American Concrete Institute, Proceedings V. 58, No. 4, Oct. 1961. Separate prints are available at 50 cents each.

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Discussion of this paper should reach ACI headquarters in triplicate by Jan. 1, 1962, for publication in Part 2, June 1962 JOURNAL.

Torres de Enfriamiento de Hormigón Armado de Forma Hiperbólica

Aqui se describen los rasgos generales, el funcionamiento y las ventajas economicas de las torres de enfriamiento de hormigón armado reforzado, de forma hiperbólica y del tipo de tiro natural.

Tours Hyperboliques de Refroidissement en Béton Armé

Ce papier décrit les caractéristiques générales, le fonctionenement et les avantages économiques des tours de refroidissement à tirage naturel, en béton armé, de forme hyperbolique.

Hyperbolische Stahlbeton-Kuehltuerme

Diese Abhandlung beschreibt die allgemeinen Merkmale, Arbeitsweise und wirtschaftlichen Vorteile der hyperbolisch geformten Stahlbeton-Kuehltuerme mit natuerlichem Zug.

Title No. 58-21

Comparison of Prestressed Concrete Beams and Conventionally Reinforced Concrete Beams Under Impulsive Loading

By G. K. WADLIN and J. J. STEWART

Pretensioned prestressed concrete beams and conventionally reinforced concrete beams of the same size were subjected to impulsive type loading and to static loading. Dynamic loads reached their peak values in from 0.005 to 0.006 sec. in a specially designed testing machine. Data were recorded with a high speed movie camera operated at speeds of about 2000 frames per sec. Comparisons of the behavior of the two types of concrete beams were based on a frame by frame analysis of the films and correlation with their static destructive tests.

■ THIS INVESTIGATION WAS an extension of a study conducted at Carnegie Institute of Technology to determine the behavior of reinforced concrete beams under impulsive loading. The results of these studies[1-3] have been published during the past 4 years. These studies indicated that the reduced ductility inherent with higher strength steel did not reduce the resistance of concrete beams reinforced with such steel to the damaging effects of impulsive loading within the range of speeds used in those tests. Because the reinforcing of prestressed concrete beams has much less ductility than even conventional hard grade steel, the influence of such little ductility on the resistance of prestressed concrete beams to impulsive loading was a subject for further investigation.

Although much is known about the behavior of prestressed concrete beams under static loads, little is known about the behavior of prestressed concrete beams under dynamic loads. An extensive search of American technical literature failed to produce any published reports on dynamic testing of pretensioned prestressed concrete beams. Magnel[4] reported in Belgium that he had performed a few impact tests on prestressed concrete beams. His method of testing consisted of dropping varying weights from different heights on the prestressed beams. He concluded qualitatively that the prestressed concrete beams did not have a high resistance to impact loads.

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ACI member **G. K. Wadlin** is professor and head, Department of Civil Engineering, University of Maine, Orono, Me. Dr. Wadlin has been associated with the University since 1948 and has also been an active structural consultant.

A modest test program was designed to study the relative strength of prestressed concrete beams and conventionally reinforced concrete beams under impulsive loading. The method of testing and the results are described in the following paragraphs.

SPECIMENS AND INSTRUMENTATION

Both types of beams had the same outside dimensions, 9 x 10 in., and 18 ft long. Both beams were designed to carry the same total static moment of 260,000 in.-lb. The total weight of each beam was slightly under 1700 lb.

Six reinforced concrete beams were cast at one time using a mix which showed a 28-day strength of 6300 psi. The reinforcement of 2.62 percent consisted of two #9 deformed bars of intermediate grade steel placed at a depth of 8½ in. from the top fiber. Fig. 1 shows the stress-strain curve for the reinforcing bars.

The prestressed concrete beams were cast ten at a time by a commercial manufacturer who employed a concrete mix that produced a

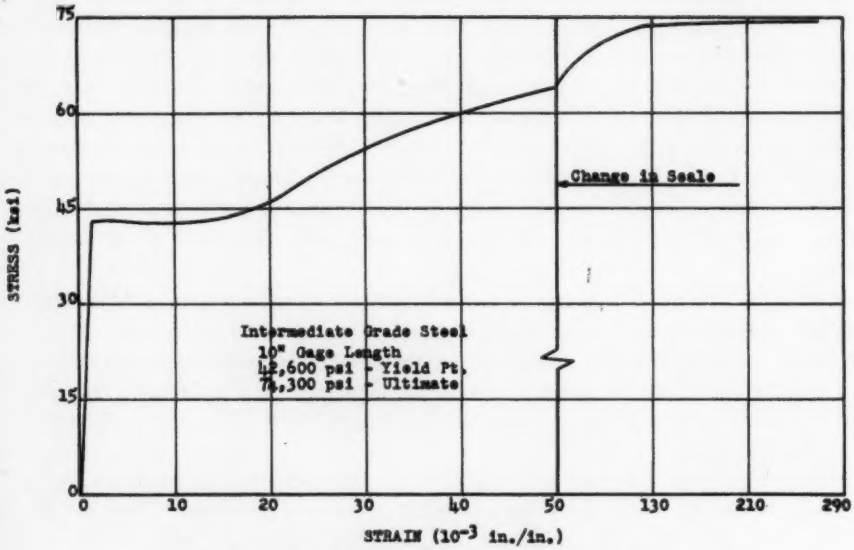

Fig. 1 — Reinforcing bar stress-strain curve

28-day strength of 7100 psi. Five 7/16 in. diameter, uncoated, stress-relieved grade strands were placed in a single layer at a depth of 7 in. from the top fiber and were initially tensioned with an 18,900-lb force per strand. Fig. 2 shows the stress-strain curve for the prestressing strand.

The beams were identified by serial numbers and letters. The first two letters in the designation described the type of concrete construction: RC for reinforced concrete or PC for prestressed concrete. The next letter described the manner of testing used: either D for dynamic testing or S for static testing. These letters were followed by a number which designated the order in which the beam was tested; hence, PCD-3 meant a prestressed concrete beam that was the third to be tested dynamically.

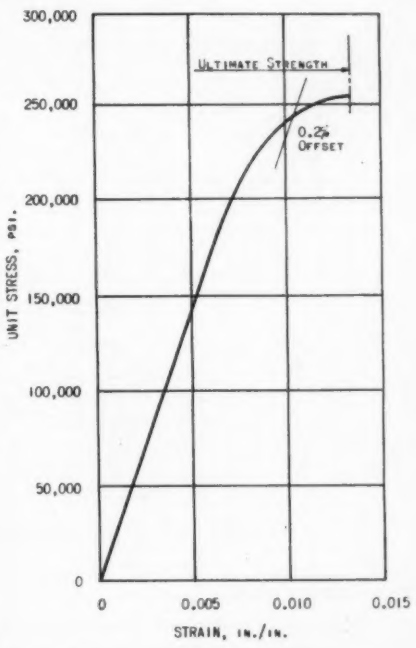

Fig. 2 — Stress-strain curve for stress-relieved grade of prestressing strand

Dynamic testing machine

The dynamic testing machine was constructed especially for this project to accommodate the size of the specimens and the predicted amount of load to be applied. The machine was designed to apply known dynamic loads up to a total of 48 kips in two equal concentrations of 24 kips at the third points of the span. The maximum loading desired was easily attained in an elapsed time of 5 millisec. The center to center distance between beam supports was 16 ft.

A schematic drawing of the dynamic testing machine is shown in Fig. 3. The machine was composed of two structural elements, the static or supporting structure, and the moving or load-applying structure. The supporting structure consisted of the base beams (A), the reaction support columns (B), the guide columns (C), the column bracing arrangement (D), and the reaction rings (E). The concrete beam specimen (F) is shown in position for testing. The structural arrangement designed to apply the dynamic loading consisted of the four load springs (G) housed in cases, the load rods (H) connecting the springs and the needle beam to the load rings (K) which applied the dynamic load directly to the beam specimen. The entire dynamic load structure was held in position by a track jack (L) prior to each test. When the jack was tripped, the load was transferred to the test beam.

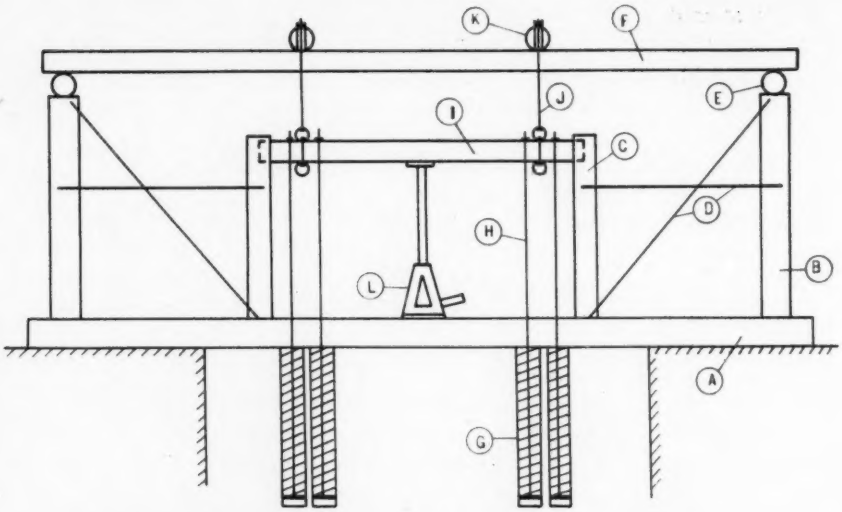

Fig. 3 — Schematic diagram of dynamic testing machine

Instrumentation

The instrumentation was designed to record all dynamic measurements on film. By superimposing all data on film, the value of each individual measurement at any instant of time was easily determined.

The right-hand load ring and reaction ring were gaged with SR-4 strain gages. Four pairs of gages on each ring were connected to the legs of a Wheatstone bridge. The unbalanced bridge voltages were fed into a dual-beam oscilloscope. Prior to the tests and during the test program, the rings were calibrated in a static compression machine.

The strains in the reinforcing bars were measured in a similar manner by mounting SR-4 gages on the bars at midspan and then noting the scope dot movement during the dynamic tests.

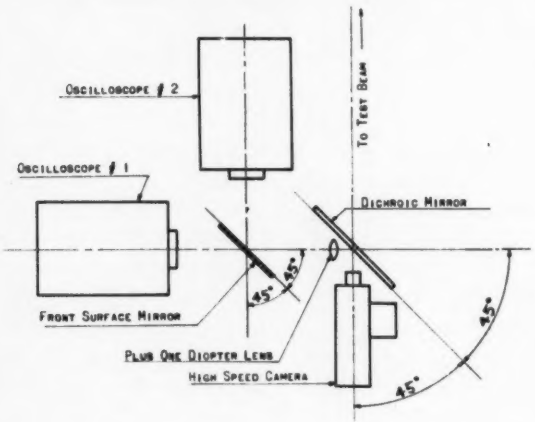

Fig. 4 — Plan view of instruments

TABLE 1 — COMPARISON OF BEAM STATIC STRENGTHS

Beam	Computed ultimate strength, kips	Actual ultimate strength, kips	Percentage difference*	Design load, kips	Safety factor†
RC-S-1	22.0	20.1	− 8.7	8.1	2.5
RC-S-2	22.0	20.6	− 6.4	8.1	2.5
PC-S-1	25.0	25.8	+ 3.2	8.1	3.2
PC-S-2	24.5	23.5	− 4.1	8.1	2.9
PC-S-3	23.0	24.4	+ 6.1	8.1	3.0

* The difference between the actual and computed ultimate strengths divided by the computed ultimate strength.
† The actual ultimate strength divided by the design load.

To superimpose the load, reaction and strain values on the film, an arrangement of mirrors was used to bring the images of the oscilloscope dots into the front lens of the high speed camera. This arrangement is shown in Fig. 4. The dichroic mirror allowed the camera to see through and record the action of the specimen while at the same time it saw the reflected images of the scope dots. A black backdrop was used behind the testing machine to secure maximum contrast for the scope dots. The initial positions of the scope dots were marked on the backdrop by pieces of aluminum foil.

The deflection measurements were obtained by mounting a horizontal steel bar on a rectangular wooden framework attached to the ceiling above the machine and then observing how far the beam moved downward below this fixed reference bar. Two black markers on the deflection reference bar were 2 ft apart. These markers provided a measure of the scale of the projected film image making it possible to scale off the deflections on the frame by frame analysis of the high speed film.

Test program

Three prestressed and two reinforced concrete beams were tested statically to destruction. Seven prestressed and four reinforced concrete beams were tested dynamically to varying degrees of failure. An attempt was made to find the dynamic load at which the beams were just on the verge of failure. This ultimate dynamic loading was then compared with the ultimate static loading of the beams in each group. In addition, the two groups were compared on the basis of their ability to carry dynamic loads.

TEST RESULTS

Static load — Deflection curves

The static load versus deflection curves shown in Fig. 5 summarize the behavior of the prestressed concrete beams and the conventionally reinforced concrete beams under static loads. The prestressed concrete beams are seen to possess higher ultimate strengths than the reinforced concrete beams. Both types of beams were reinforced for the same static design moment. The reinforced concrete beams were designed according to the ACI Code (ACI 318-56) and the prestressed concrete beams were designed according to the code adopted by the Pennsyl-

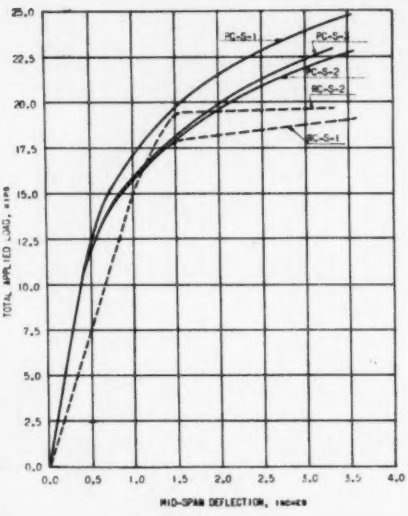

Fig. 5 — Static load versus midspan deflection

vania Department of Highways. Because the prestressed concrete design code was more conservative than the conventional reinforced concrete design code, the prestressed concrete beams possessed higher ultimate strengths than the conventionally reinforced concrete beams. Table 1 shows that the safety factor was approximately 2.5 for the reinforced concrete beams and 3.0 for the prestressed beams.

Prestressed concrete beams are seen in Fig. 5 to be considerably stiffer than the conventionally reinforced concrete beams. Spring stiffness factors were 15.0 kips per in. for the reinforced concrete beams and 24.6 kips per in. for the prestressed concrete beams.

Dynamic load — Reaction versus time curve

Data provided by the film record of the dynamic tests were used to plot values of total loads and total reaction versus time. Fig. 6 shows a typical curve of load and reaction versus time for a reinforced concrete beam. Fig. 7 shows the same curves for prestressed concrete beams. The over-all pattern of applied loads and reactions appears to be quite similar for both types of beams.

As can be seen from an examination of these curves, values of dynamic load increased rapidly initially to their peak values. The dynamic loads decreased from their peak values as the test beam was accelerated downwards from beneath the load ring and increased again as the load ring "caught up" with the beam. The load then oscillated as the beam continued to vibrate. Consequently, the test beams were subjected to cycles of dynamic loading of varying magnitude rather than "one shot" loading. The magnitude of the applied dynamic load at any instant depended on the stiffness of the components of the testing machine, particularly the springs, as well as on the stiffness of the test specimens. Because the two types of beams had different stiffnesses, it was apparent that testing of the two types of beams under identical forces stored in the springs would not result in identical impulses applied to both. Instead, comparisons were based on the peak values of dynamic load actually imposed on the beams as measured by

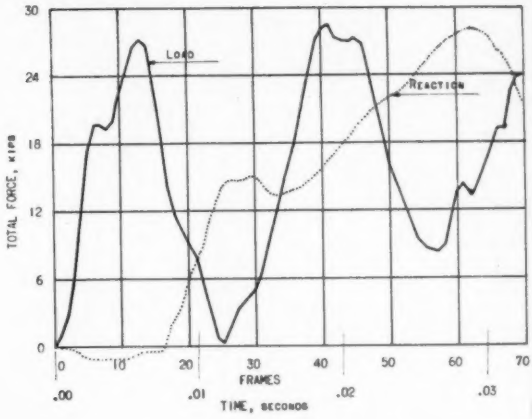

Fig. 6 — Load-reaction versus time (Beam RC-D-2)

strain gages and oscilloscopes and on the impulse actually delivered to each beam as measured by the area under the load versus time curves.

The greater stiffness of the prestressed concrete beams is evident from the steeper slopes of the load versus time curves for the prestressed concrete beams. The peak values of the initial impulse were obtained in 0.0064 and 0.0051 sec after the beginning of the loading cycle for the conventionally reinforced and prestressed concrete beams, respectively, and appeared to be independent of the magnitude of the peak dynamic load. The time required for the peak load to decrease to zero was slightly less than the time required for the loading to increase initially from zero to the peak value for the reinforced concrete beams. The duration of the first impulse was slightly longer for the reinforced concrete beams so that although the peak load was reached earlier on the prestressed concrete beams, the areas under the

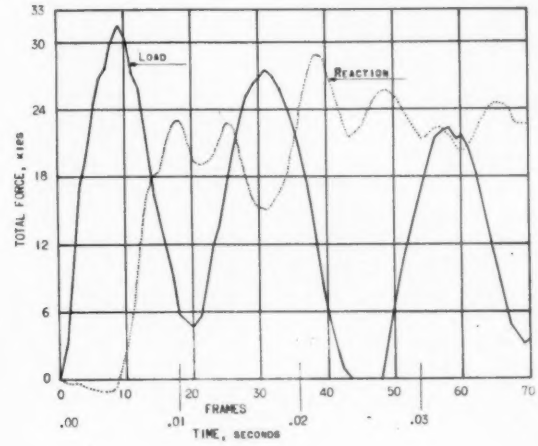

Fig. 7 — Load-reaction versus time (Beam PC-D-7)

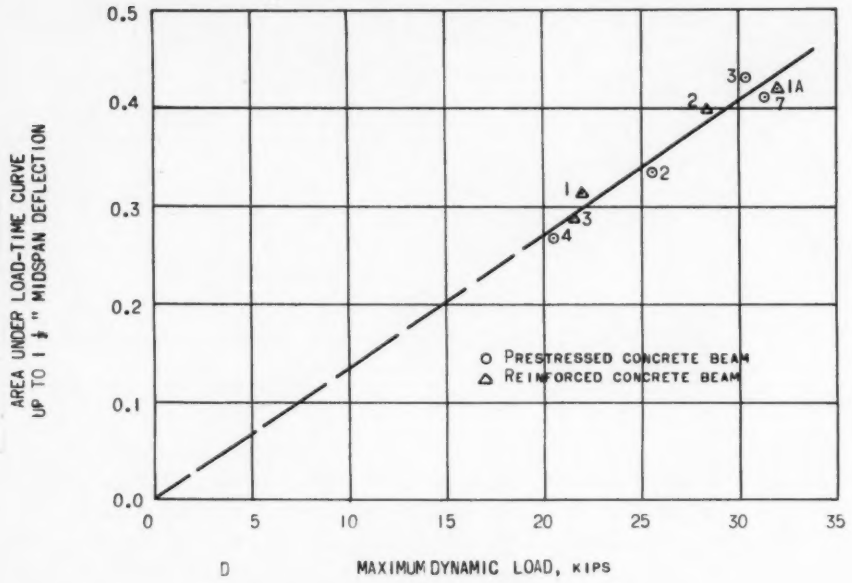

Fig. 8 — Impulse versus dynamic load

loading curves were approximately equal for the first impulse under equal peak loads. The ratio of the time required to reach maximum dynamic load to the natural period of the beam was 0.061 for the reinforced concrete beams and 0.065 for the prestressed concrete beams. Consequently, the severity of the impulsive loading was approximately the same for both types of beams.

The reaction versus time curves shown in Fig. 6 and 7 indicate that a definite period elapsed between the initial application of the load and the initial positive increase in the reaction. Typical of high speed dynamic tests, the reaction was a negative value immediately after the initial application of the load. This negative value never became as large as half of the test beam dead load so that the beam was always in contact with the reaction ring. The higher stiffness of the prestressed concrete beams was manifested again in the shorter time required for the reaction to have a positive value. Furthermore, the reactions of the prestressed concrete beams increased at a more rapid rate than that of the conventionally reinforced concrete beams. After reaching a maximum value the reaction oscillated up and down out of phase with the load value until the beam ceased to accelerate up and down or else failed completely.

The impulses absorbed by both types of beams were approximately equal during the initial stages of deflection. Fig. 8 shows the area under

the load-time curve plotted against the maximum dynamic load applied for each beam. The areas shown were measured until a midspan deflection of 1½ in. had been obtained. The curve indicated that the impulse required to deflect the beams at midspan 1½ in. was a linear function of the applied peak dynamic load. No differences between the two types of beams are indicated.

The time required to reach dynamic yielding of the reinforcement varied from 0.022 to 0.029 sec. This is well within current concepts of the time required for members of building frames to reach yield stresses under blast loading.

Loads and deflections

The dynamic deflections at midspan of the prestressed concrete beams and the conventionally reinforced concrete beams were compared on the basis of their design and ultimate strengths. Both types of beams were designed for identical loads. As discussed earlier, this led to differences between the ultimate strengths of the two types of beams. Furthermore, because the distance from the top of the prestressed concrete beams to the centroid of the prestressed cables varied significantly in all test specimens, the ultimate strengths of all the prestressed concrete beams differed from each other. Consequently, because of the varying ultimate strength of each prestressed specimen, comparisons of behavior based solely on the values of the maximum applied dynamic loads were unrealistic. However, the ratio of the maximum dynamically applied loads P_d to the computed static ultimate strength P_{su} did provide a true indication of the relative intensity of each loading. Final midspan deflections of the test beams are plotted in Fig. 9 against the ratios of P_d/P_{su}. For indentical values of P_d/P_{su}, the final midspan deflections of the prestressed concrete beams exceeded those of the conventionally reinforced concrete beams.

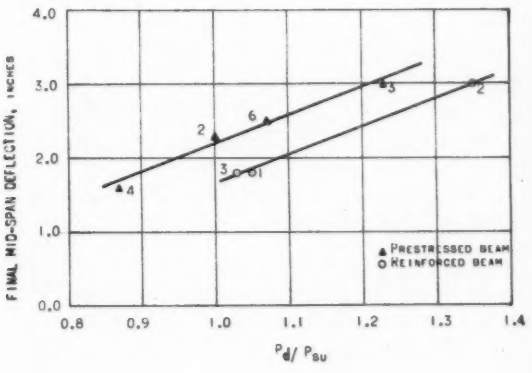

Fig. 9 — Final midspan deflection versus P_d/P_{su}

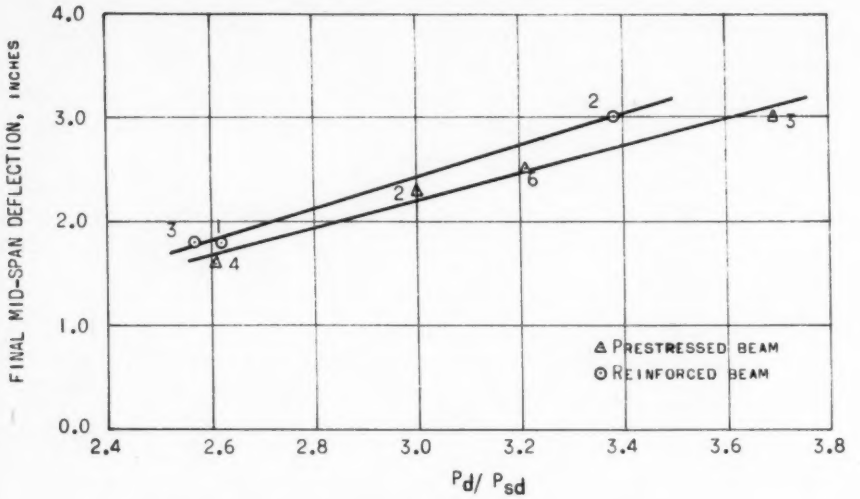

Fig. 10 — Final midspan deflection versus P_d/P_{sd}

If the midspan deflections are compared with the ratio of the maximum applied dynamic load P_d to the static design load P_{sd}, the positions of the curves relative to each other in Fig. 9 are reversed. This is shown in Fig. 10 and is explained by the larger safety factors required for prestressed concrete beams. Using the ratio P_d/P_{sd} as a basis of comparison, conventionally reinforced concrete beams deflected more than the prestressed beams.

Residual loads

The test beams that did not fail completely came to rest with some force still in the springs. The value of the force remaining in the springs, hereafter referred to as the residual load, was obtained by comparing the height of the needle beam above the machine base with a calibration curve of needle beam height versus spring load. Obviously, after having been subjected to some value of dynamic load, each test beam would come to rest only after the condition for static equilibrium had been established; that is, the remaining static load carrying capacity of the beam was equal to the force in the springs. Each test beam would continue to deflect until this condition was satisfied. Consequently, the value of the residual load was a good quantitative measure of the structural damage that each beam had suffered.

A comparison of residual loads versus P_d/P_{sd} for the two types of beams, Fig. 11, dramatizes the superior resistance to dynamic loading of the prestressed beams when compared with reinforced beams on the basis of their static design loads. Fig. 11 shows that the residual

load carried by the prestressed beams increased with increasing values of P_d/P_{sd} to 15.8 kips when P_d/P_{sd} was 3.69. When the applied dynamic load was 3.78 times as large as the static design load, the residual load was 10.8 kips. This drop in the load carried after the test indicated that the beam had partially failed. The concrete had crushed at the top of this beam to a depth of 1½ in. When P_d/P_{sd} equaled 4.26, the prestressed beam failed completely. The maximum dynamic load that the prestressed beams could carry without any degree of failure was not determined precisely but a study of Fig. 11 indicates that the critical value of P_d/P_{sd} is somewhere between 3.69 and 3.78. The minimum load that would have caused complete failure (zero residual load) probably is considerably less than the value of 4.26 times the static design load that completely destroyed Beam PC-D-7.

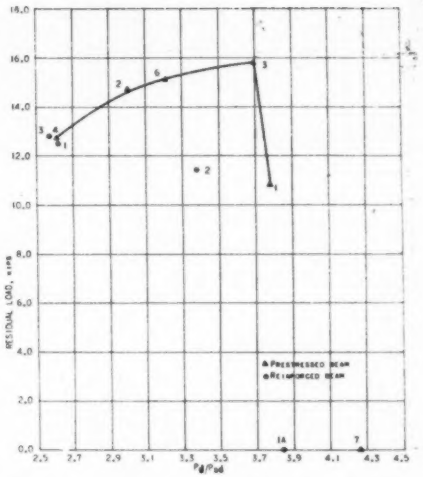

Fig. 11 — Residual load versus P_d/P_{sd}

Not enough tests were conducted on the reinforced concrete beams to permit drawing a curve through the points shown in Fig. 11. The curve probably increases from Points 1 and 3 with increasing values of P_d/P_{sd} in some manner similar to the prestressed beam curve. However, the ratio of P_d/P_{sd} at which the residual load decreases (partial failure) is considerably less than the critical values of P_d/P_{sd} for the prestressed beams. Point 2 indicates that for a value of P_d/P_{sd} of 3.38, the reinforced concrete beam had begun to fail, carrying a residual load of 11.4 kips, although a prestressed beam would have been sound and capable of carrying a residual load of approximately 15.5 kips. The minimum dynamic load that would have caused complete failure of the reinforced beam probably is much less than 3.83 times as large as the static design load indicated by Point 1A in Fig. 11. It is interesting to note here that for similar values of P_d/P_{sd} (3.78 and 3.83) the prestressed beam failed only slightly but the reinforced concrete beam failed completely.

A comparison of residual load versus P_d/P_{su} would result in a curve similar to that in Fig. 11 except that the abscissas of the prestressed beam points and the reinforced beam points would be 1/3.0 and 1/2.5, respectively, times the values shown in Fig. 11. The effect is to make the reinforced beam appear slightly better than the prestressed beam.

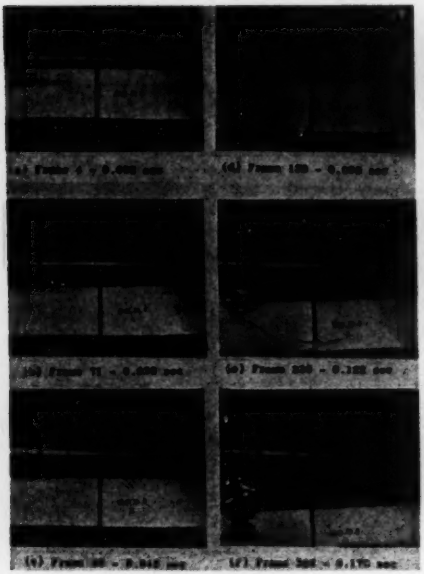

Fig. 12 — Dynamic failure of a reinforced concrete beam

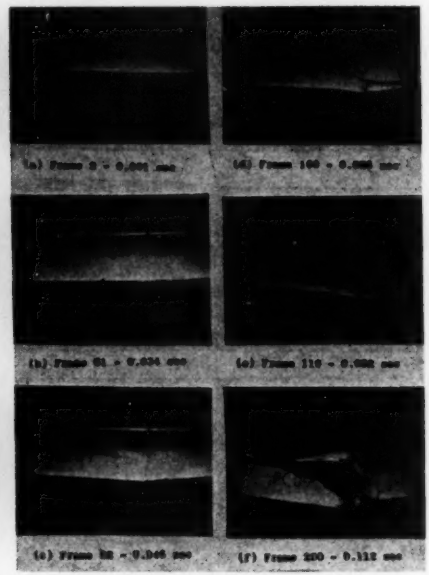

Fig. 13 — Dynamic failure of a prestressed concrete beam

For example, with P_d/P_{su} equal to 1.26 and 1.35 for the prestressed beam and reinforced beam, respectively, the residual load was 10.8 kips for the prestressed beam and 11.4 kips for the reinforced beam.

Types of failure

Two partial failures and three complete failures occurred in the total of 11 dynamic tests. The partial failures occurred within the middle third of one prestressed and one reinforced concrete beam.

Two of the complete failures, one of each beam type, can be seen in Fig. 12 and 13 which were taken from the high speed movie film record. The two beams had approximately equal static ultimate strengths and were subjected to approximately equal dynamic loads. It was interesting to note that approximately equal amounts of energy, as measured by the area under the dynamic load-deflection curves, were required to initiate crushing in the compression zone for both types of beams. This initial crushing was followed by complete shattering of the beams. The prestressed concrete beam disintegrated in about half the time it took for the reinforced concrete beam to reach the same state. This action is clearly seen in Fig. 12 and 13.

TABLE 2 — MAXIMUM DYNAMIC LOADS AND MIDSPAN DEFLECTIONS

Beam	Maximum dynamic load, kips	Maximum midspan dynamic deflection, in.	Final midspan deflection, loaded, in.	Final midspan deflection, unloaded, in.
RC-D-1	22.0	1.8	1.8	1.8
RC-D-1A	32.1	∞	—	—
RC-D-2	28.4	3.6	3.0	3.0
RC-D-3	21.6	1.8	1.8	1.8
PC-D-1	27.0	3.2	2.7	0.9
PC-D-2	25.6	2.3	2.3	0.2
PC-D-3	30.4	3.0	3.0	0.5
PC-D-4	20.5	1.6	1.6	0.2
PC-D-6	26.0	2.5	2.5	0.6
PC-D-7	31.4	∞	—	—

Recovery after removal of load

When the loading frame was removed after the tests, the prestressed beams, except for the beam that failed completely, rebounded almost to their original positions. The reinforced concrete beams rebounded insignificant amounts. In some instances, the prestressed beams still had some camber after the load was removed. These data are shown in Table 2. Cracks which had extended from the bottom to within 2½ in. from the tops of the prestressed beams disappeared almost completely when the residual load was removed.

SUMMARY

For the range of speeds, 7000 kips per sec to 9200 kips per sec, and the range of loads used in this test program, the following was observed:

1. The prestressed concrete beams behaved as stiffer beams than the conventionally reinforced concrete beams under both static and dynamic loading.

2. Whether the reinforced or prestressed concrete beams offered more resistance to impulsive loading depended on whether they were compared on the basis of static ultimate loads or on the basis of static design loads permitted under present codes. All beams were reinforced for the same static design load. However, because existing codes for prestressed concrete are more conservative than reinforced concrete codes, prestressed beams had a higher ultimate strength than the reinforced concrete beams.

(a) When compared on the basis of design strength the prestressed beams deflected less and suffered less structural damage as measured by their ability to carry residual loads than the reinforced concrete beams under similar ratios of P_d/P_{sd}. When P_d/P_{sd} was 3.83 for the

reinforced concrete beams, complete failure occurred. When P_d/P_{sd} was 3.78 for the prestressed beams, only partial failure occurred. In this partially failed condition, the prestressed concrete beam carried a residual load of 10.8 kips. The numerical values of these ratios have significance only for the beams used in this test program and for the test conditions described. However, they do afford a basis for comparing the resistance of these two types of concrete beams to impulsive loading.

(b) When compared on the basis of ultimate strength, the differences between the behavior of the reinforced and prestressed concrete beams under impulsive loading were small. The reinforced concrete beams were slightly superior.

3. In the limited number of tests to complete destruction:

(a) The reinforced and prestressed beams which had almost identical static ultimate strengths absorbed approximately equal amounts of energy before crushing of the concrete occurred.

(b) After the concrete began to crush, the prestressed beam failed completely in half the time required for the reinforced concrete beam.

(c) No reinforcing bars or prestressed strands were broken. The limited ductility of the prestressing steel was of no significance in these tests.

4. The prestressed concrete beams outperformed the conventionally reinforced concrete beams in their ability to regain their original geometry and appearance. On removal of the residual loads, prestressed concrete beams rebounded almost to their initial positions. Some even regained camber. Cracks that had opened during dynamic testing closed on removal of the residual loads. The conventionally reinforced concrete beams, however, after having been tested under similar dynamic loads, rebounded insignificant amounts.

ACKNOWLEDGMENTS

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The authors wish to thank the Dickerson Structural Concrete Corp., Youngwood, Pa., for their donation of the prestressed concrete beams used in these tests.

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Comparacion Entre Vigas de Hormigón Pretensado y Vigas de Hormigón Armado Convencionales Bajo Cargas Impulsivas

Se sometieron a cargas impulsivas y estáticas, vigas de la misma medida, de hormigón preestirado y pretensado y vigas de hormigón armado convencionales. Los valores máximos de cargas dinámicas se obtuvieron de 0.005 á 0.006 segundos en máquina destinada expresamente para esta prueba. Los datos obtenidos se registraron con una cámara cinematográfica de alta velocidad operada a razón de 2000 cuadros por segundo. Las comparaciones en el comportamiento de los dos tipos de viga se basaron y obtuvieron en un análisis de cuadro por cuadro de la película y sus relaciones con las pruebas estáticas destructivas.

Comparaison des Poutres de Béton Précontraint et des Poutres de Béton Armé Sous une Charge Impulsive

Les poutres de béton précontraint avec fils préalablement tendus et les poutres de béton armé de la même dimension, ont été assujetties à une charge du type impulsif et à une charge statique. Les charges dynamiques atteignient ses points de valeur de 0.005 à 0.006 de la seconde, dans une machine d'essai spécialement dessinée. Les données etaient registrées avec une cinécaméra de haute vitesse, opérée à une vitesse de 2000 cadres par la seconde. Les comparaisons de la conduite des poutres de deux types de béton etaient basées dans une analyse de cadre à cadre des pellicules et la corrélation avec ses essais destructifs statiques.

Vergleich zwischen Spannbetontraegern und herkoemmlichen Eisenbetontraegern unter schwellender Belastung

Vorgespannte Spannbetontraeger und herkoemmliche Eisenbetontraeger gleicher Groesse wurden schwellenden und statischen Belastungen unterworfen. In einer besonders konstruierten Pruefmaschine erreichten dynamische Belastungen ihre Spitzenwerte innerhalb von 0,005 bis 0,006 sec. Die Angaben wurden von einer schnellaufenden Kamera aufgenommen, die mit einer Geschwindigkeit von 2000 Bildern pro Sekunde arbeitet. Vergleiche in dem Verhalten der beiden Betontraegerarten wurden begruendet mit einer "Bild fuer Bild" Analyse der Filme und der Wechselbeziehung mit ihren statischen Bruchversuchen.

Title No. 58-22

Design of the Continuous Arched Frame Supporting Cylindrical Shells

By ALFRED ZWEIG

Presents a method for designing continuous arched frames and offers tables for use with the most commonly used loadings, and frame shapes. The method will allow design of these frames, which normally require the simultaneous solution of a number of equations, without great effort. An appendix provides the mathematical derivation of the formulas offered.

■ THE EVER-INCREASING USE of cylindrical shell roofs confronts the designer more and more with the necessity of designing continuous arched frames as end support for the shells. Since the Hardy Cross method is not easily applicable to this type of frame, a solution of this problem will necessitate solving n simultaneous equations with n unknowns, where n represents the number of bays of the continuous frame. This is a rather tedious task which is especially complicated by the fact that most coefficients for these equations are obtained as small differences of large quantities and, consequently, these values can be ascertained with any degree of accuracy only by the use of a calculating machine or other time-consuming methods.

Accuracy to this extent requires more time and effort than the average engineer is willing to invest; and the purpose of this paper is to present a design method and design tables for the most commonly used loadings and frame shapes which permit design of such frames without great effort. The mathematical derivation of all formulas is omitted from the general description and confined to the Appendix only.

GENERAL DESCRIPTION

The shape of the frame to be analyzed is shown in Fig. 1a. It consists of n arches with radius r, rigidly connected with each other and with the two end columns. The arch thrust is taken at the exterior columns only, with the interior columns assumed not to be connected rigidly with the arch but only to provide vertical support. This assumption is well justified since, even in the case of a rigid connection between arches and interior columns, these columns will take only

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a negligible amount of the arch thrust if both of the adjacent arches are equally loaded and if the exterior columns are much stiffer than the interior ones.*

The exterior column bases are assumed to be hinged and the arch thrust can be either resisted by the soil at the exterior foundation or, where the soil does not lend itself to the resistance of the horizontal forces, the exterior foundations at each end are held together with a tie rod, the latter being a more commonly encountered detail (see Fig. 1b).

This frame, if analyzed by the method of redundant forces, is statically indeterminate to the nth degree.

To avoid the tedious job of solving a large number of simultaneous equations, a somewhat different approach will be offered in this paper which makes use of the method of difference equations and which is based on the following thought:

To analyze the frame shown in Fig. 1a, we will remove, for the time being, the horizontal support on one column base and establish moments and reactions on the resulting system, which is a continuous beam without arch action. This continuous beam, instead of being made up of straight members, is made up of curved members. For its solution, we will use the well-known three-moment theorem which establishes a relationship between the three successive moments $X_k{}^c{}_{-1}$, $X_k{}^c$, $X_k{}^c{}_{+1}$, at Supports $K-1$, K, and $K+1$, respectively. This leads to $(n-1)$ equations for the $(n-1)$ unknown moments at the supports.

Instead, however, of solving these equations in the familiar way by successive elimination, we will, rather, regard the three-moment equation as a difference equation† for which the theory of difference equations furnishes us with a direct solution in such a way that any moment $X_k{}^c$ at the Support k may be written as a simple function of the loading, shape, and number (n) of bays. This will completely eliminate the necessity for the time-consuming task of solving $(n-1)$ equations for the $(n-1)$ unknowns, though the results of the difference equations are identical with those given by the solution of the simultaneous

*For further justification of this assumption, see also the special study in connection with the practical example given later.

†For other problems solved by this method, see Parme, Alfred, "Solution of Difficult Structural Problems by Finite Differences," ACI JOURNAL, *Proceedings* V. 47, No. 3, Nov. 1950, pp. 237-256.

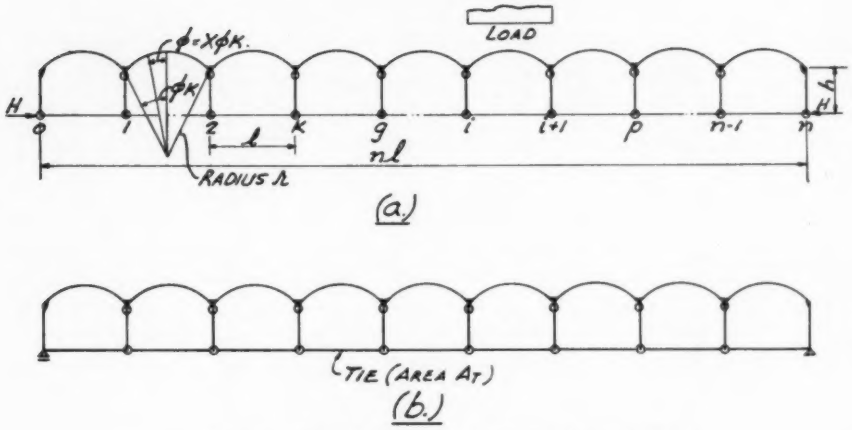


Fig. 1—Typical arched frame under analysis

equation. The results obtained by use of difference equations have, however, the additional advantage that they may be written in a direct and explicit form and that they lend themselves, therefore, particularly well to the derivation of generally applicable formulas.

Such formulas have been developed in the Appendix and are tabulated in the first column of Table 2 for the practical use of the designer.

It should be noted that the solution for the various loadings depends on a set of coefficients $\overline{k}_3$, $\overline{k}_6$ through $\overline{k}_9$ and k_{15} through k_{20}, and on the auxiliary function $K_{k(n)}$, the values for which are tabulated in Tables 3 and 4. Function $K_{k(n)}$ approaches a constant value with increasing the number of bays and it is, therefore, sufficiently accurate for $n > 6$ to use the values for $n = 6$. The moments on the continuous beam can now be found by combining the influence of the moments over the support from Table 2 with the influence of the load on the noncontinuous beam. Table 1 gives the moments for the various loadings on the noncontinuous beam on two supports.

Having thus solved the continuous beam over n supports, we will now determine the horizontal thrust through the following consideration:

> After removing the horizontal support at the column base and loading the continuous beam with the given loading L, the laterally unsupported column base will move laterally a distance D_L as shown in Fig. 2a.

Since the laterally supported column base will not permit such a movement, we have to apply a horizontal thrust H of such magnitude and direction that the horizontal deflection D_H of the column base, due to this thrust, will equal the deflection D_L due to the load-

TABLE I — MOMENTS M_ϕ FOR DISCONTINUOUS ARCHED BEAMS

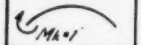
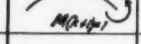
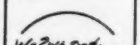
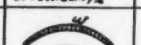
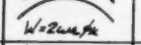
LOADING	EXACT VALUE	VALUE OBTAINED BY MEANS OF A POWER SERIES	AT $\phi = \frac{\phi_k}{2}$	$\phi = 0$	$\phi = -\frac{\phi_k}{2}$
1 	$\frac{\sin\phi_k + \sin\phi}{2\sin\phi_k}$	$\frac{1+X}{2} + \frac{\phi_k^2}{12}(x-x^3)$	$\frac{3}{4}\left(1+\frac{\phi_k^2}{24}\right)$	$1/2$	$\frac{1}{4}\left(1-\frac{\phi_k^2}{8}\right)$
2	$\frac{\sin\phi_k - \sin\phi}{2\sin\phi_k}$	$\frac{1-X}{2} - \frac{\phi_k^2}{12}(x-x^3)$	$\frac{1}{4}\left(1-\frac{\phi_k^2}{8}\right)$	$1/2$	$\frac{3}{4}\left(1+\frac{\phi_k^2}{24}\right)$
3 	$k(\cos\phi - \cos\phi_k)$	$k\left[\frac{\phi_k^2}{2}(1-x^2) - \frac{\phi_k^4}{24}(1-x^4)\right]$	$\frac{3}{32}k\left[\phi_k^2 - \frac{5}{48}\phi_k^4\right]$	$\frac{k}{2}\left[\phi_k^2 - \frac{\phi_k^4}{12}\right]$	
4	$\frac{W_k}{4}\left(\sin\phi_k - \frac{\sin\phi}{\sin\phi_k}\right)$	$\frac{W_k}{4}\left[\phi_k(1-x^2) - \frac{\phi_k^3}{6}(1+x^2-2x^4)\right]$	$\frac{3}{16}W_k\left[\phi_k - \frac{1}{4}\phi_k^3\right]$	$\frac{W_k}{4}\left(\phi_k - \frac{\phi_k^3}{6}\right)$	
5	$\frac{W_k}{2}\left(\sin\phi_k + \frac{\cos\phi_k}{\phi_k} - \phi\frac{\sin\phi}{\phi_k} - \frac{\cos\phi}{\phi_k}\right)$	$\frac{W_k}{4}\left[\phi_k(1-x^2) - \frac{\phi_k^3}{4}(1-x^4)\right]$	$\frac{3}{16}W_k\left[\phi_k - \frac{5}{16}\phi_k^3\right]$	$\frac{W_k}{4}\left(\phi_k - \frac{\phi_k^3}{4}\right)$	
6 	$\frac{W_k}{\phi_k}\left[\cos\phi + \phi_k\sin\phi_k - \frac{\sin\phi_k\cos\phi_k}{\phi_k} - \phi^2\frac{\sin\phi_k}{2\phi_k} - \frac{\phi\sin\phi}{2} + \frac{\sin^2\phi_k\cos\phi_k}{2} - \cos\phi\left(1 + \frac{\sin^2\phi_k}{2} + \frac{\sin\phi_k\cos\phi_k}{2\phi_k}\right)\right]$	$\frac{W_k}{48}\left[\phi_k(11-15x^2+5x^4-x^6) - \frac{\phi_k^3}{24}\frac{225-165x^2-45x^4+15x^6-3x^8}{24}\right]$	$\frac{3}{16}W_k\left[\frac{225}{384}\phi_k - \frac{1485}{4608}\phi_k^3\right]$	$\frac{W_k}{4}\left(\frac{11}{8}\phi_k - \frac{235}{672}\phi_k^3\right)$	
7 	$\frac{P_k}{2}(\sin\phi_k - \sin\phi)$	$\frac{P_k}{2}\left[\phi_k(1-x) - \frac{\phi_k^2}{6}(1-x^2)\right]$	$\frac{P_k}{4}\left[\phi_k - \frac{7}{24}\phi_k^3\right]$	$\frac{P_k}{2}\left(\phi_k - \frac{\phi_k^3}{6}\right)$	
8 	$\frac{W_k}{4}(\sin\phi_k - \sin\phi)$ $\phi_k \geq \phi \geq 0$ $\frac{W_k}{4}\left(\sin\phi_k - \sin\phi - \frac{2\sin^2\phi}{\sin\phi_k}\right)$ $0 \geq \phi \geq -\phi_k$	$\frac{W_k}{4}\left[\phi_k(1-x) - \frac{\phi_k^3}{6}(1+x^2)\right]$ $\frac{W_k}{4}\left[\phi_k(1-x-2x^2) - \frac{\phi_k^3}{6}(1+2x^2+x^4-4x^6)\right]$	$\frac{W_k}{4}\left[\frac{\phi_k}{2} - \frac{7}{48}\phi_k^3\right]$	$\frac{W_k}{4}\left(\phi_k - \frac{\phi_k^3}{6}\right)$	$\frac{W_k}{4}\left[\phi_k - \frac{7}{48}\phi_k^3\right]$

TABLE 2 — MOMENTS AND HORIZONTAL DEFLECTIONS FOR VARIOUS LOADINGS

LOADING	MOMENT $X_x^{(n)}$ AT SUPPORT (x) FOR CONTINUOUS BEAM	HORIZONTAL DEFLECTION D AT BASE
1	$X_x^{(n)} = -\frac{h}{30}[30-(30+10m\,\eta_x^2-m\eta_x^5)K_{x(n)}]$ $= -h[1-(1+m\bar{k}_x)K_{x(n)}]$ where $m=\frac{h}{t}$	$EI_0D = h^3(mk_0+2Sk_0+Z)$ $S=\frac{t_{(n)}}{t_{(n)}}-\bar{k}_0(1+\bar{k}_0)$ $Z=\frac{t_0}{t_{(n)}}+\frac{2k_0}{t_{(n)}}(\frac{t_0}{t_{(n)}}+k_0)$
2	$X_x^{(n)} = W_0(-\frac{\eta_x}{6}+\frac{1}{20}\eta_x^3)K_{x(n)} = -W_0\bar{k}_0K_{x(n)}$	$EI_0D = -Wh^3(mk_0+2Sk_0)$
3	$X_x^{(n)} = W_0(-\frac{\eta_x}{6}+\frac{1}{20}\eta_x^3)K_{x(n)} - W_0\bar{k}_1K_{x(n)}$	$EI_0D = -Wh^3(mk_0+2Sk_1)$
4	$X_x^{(n)} = W_0(-\frac{\eta_x}{6}+\frac{3}{20}\eta_x^3)K_{x(n)} = -W_0\bar{k}_2K_{x(n)}$	$EI_0D = -Wh^3(mk_0+2Sk_2)$
5	$X_x^{(n)} = P(-\frac{1}{6}\eta_x+\frac{1}{20}\eta_x^3)K_{x(n)} = -W_0\bar{k}_0K_{x(n)}$	$EI_0D = -P_0h^3(mk_0+2Sk_0)$
6	$X_y^{(n)} = B^i_{y(n)}\times\frac{W_0}{2}(-\eta_x+\frac{1}{3}\eta_x^3) = -B^i_{y(n)}W_0K_{x(n)}$ $X_x^{(n)} = B^i_{x(n)}\times\frac{W_0}{2}(-\eta_x+\frac{1}{3}\eta_x^3) = -B^i_{x(n)}W_0K_{x(n)}$	$EI_0D = -Wh^3(k_0+k_0T)\frac{T+\frac{1}{12}-(\frac{1}{12}+k_0)(k_{1(n)}+k_{(n)})}{}$
7	$X_y^{(n)} = B^i_{y(n)}\times\frac{W_0}{2}(-\eta_x+\frac{1}{20}\eta_x^3) = -B^i_{y(n)}W_0K_{x(n)}$ $X_x^{(n)} = B^i_{x(n)}\times\frac{W_0}{2}(-\eta_x+\frac{1}{20}\eta_x^3) = -B^i_{x(n)}W_0K_{x(n)}$	$EI_0D = -Wh^3(k_1+k_1T)$
8	$X_y^{(n)} = B^i_{y(n)}\times\frac{W_0}{2}(-\frac{7}{9}\eta_x+\frac{487}{1620}\eta_x^3) = -B^i_{y(n)}W_0K_{x1}$ $X_x^{(n)} = B^i_{x(n)}\times\frac{W_0}{2}(-\frac{7}{9}\eta_x+\frac{487}{1620}\eta_x^3) = -B^i_{x(n)}W_0K_{x1}$	$EI_0D = -Wh^3(k_1+k_1T)$
9	$X_y^{(n)} = B^i_{y(n)}\times\frac{P_0}{2}(-3\eta_x+\frac{5}{20}\eta_x^3) = -B^i_{y(n)}P_0K_{x0}$ $X_x^{(n)} = B^i_{x(n)}\times\frac{3}{2}(-3\eta_x+\frac{5}{20}\eta_x^3) = B^i_{x(n)}\cdot P_0K_{x0}$	$EI_0D = -Wh^3(k_0+k_0T)$
10	$X_y^{(n)} = -W_0[P_{x_y}B^i_{y(n)}+k_{x_0}\bar{B}^{(n)}_{y(n)}]$ $X_x^{(n)} = -W_0[k_{x_y}\bar{B}^{(n)}_{x(n)}+k_{x_0}\bar{B}^{(n)}_{x(n)}]$	$EI_0D = -Wh^3(k_{x_0}+G_{(1)}k_{x_{(n)}}+G_{(n)}k_{x_{(n)}})$ $G_{(1)}=\frac{1}{12}-(\frac{1}{12}+k_0)K_{1(n)}$ $G_{(n)}=\frac{1}{12}-(\frac{1}{12}+k_0)K_{(n)(n)}$
	For: $0\le\eta\le i$ For: $i+1\le\eta\le n$ $B^i_{y(n)}=\frac{\beta^{n-i}-\beta^{n-i}}{\beta^n-1}(\beta^n-\beta^{-n})$ $B^i_{x(n)}=\frac{\beta^{n-n}-\beta^n}{\beta^n-1}(\beta^{(n)}-\beta^{-n})$	
	$\bar{B}^{(i)}_{y(n)}=\frac{B(\beta^n-\beta^{-n})}{1-\beta^{2n}}(\beta^n-\beta^{2n-i})$ $\bar{B}^{(i+1)}_{y(n)}=\frac{B(\beta^n-\beta^{2n-2(i+1)})}{1-\beta^{2n}}(\beta^{(i)}-\beta^{2n-2(i+1)})$	$\bar{B}^{(i)}_{x(n)}=\frac{B(\beta^n-\beta^{2n-n})}{1-\beta^{2n}}(\beta^{(i)}-\beta^{-(i)})$ $\bar{B}^{(i+1)}_{x(n)}=\frac{B(\beta^n-\beta^{2n-n})}{1-\beta^{2n}}(\beta^{(i+1)}-\beta^{-(i+1)})$

ing, and be in opposite direction to it or $D_L + D_H$ must be zero. Applying a unit load $H = 1$ at the column base (see Fig. 2b), we first find $D_{H=1}$, and from this, the deflection D_H can easily be found by multiplying H by $D_{H=1}$, from which the basic equation to determine the horizontal thrust H follows:

$$H = -\frac{D_L}{D_{H=1}} \quad \text{(1)}$$

Deflection values for various loadings have been derived in the Appendix and the last column in Table 2 gives a summary of these values.

Where a tie rod is used, such as shown in Fig. 1b, the elongation of the tie, $D_T = nl/E_T A_T$, should be added to the expression $D_{H=1}$ in Table 2 where E_T represents the modulus of elasticity and A_T the cross-sectional area of the tie.

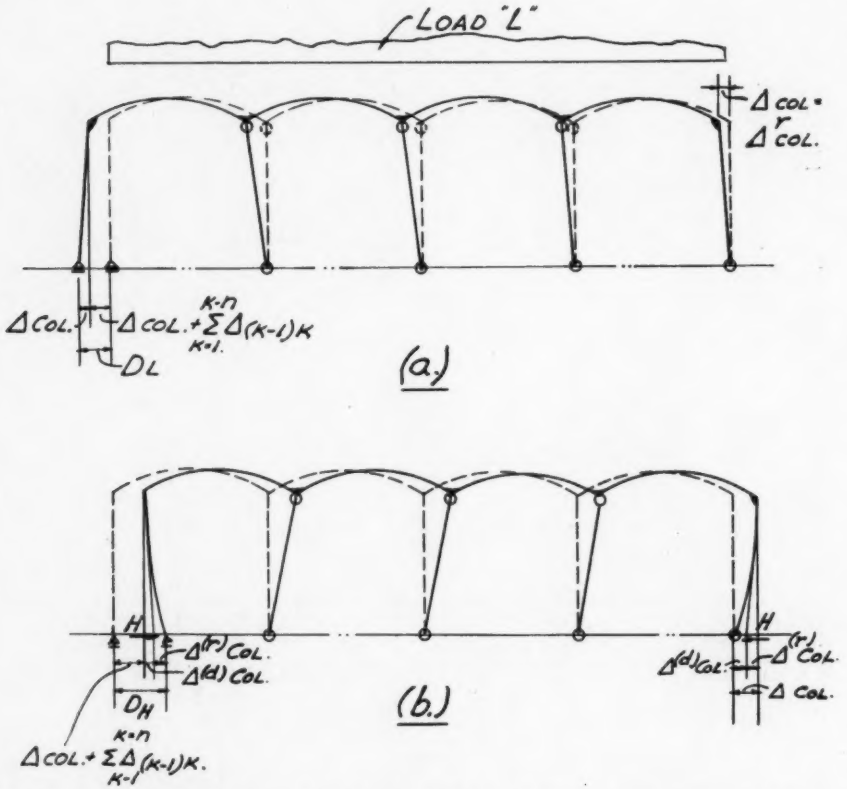


Fig. 2 — Frame deflections; (a) due to Load L; (b) due to Force H at base of exterior column

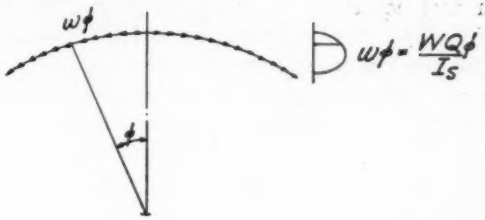


Fig. 3—Distribution of shear load

Expressions for the deflection D depend on a set of coefficients k_3 through k_{14} and, in those cases where all bays are loaded, on the auxiliary function $A_{(n)}$, the values for which are tabulated in Tables 3 and 4. It is noteworthy that with an increasing number of bays, $A_{(n)}$ approaches a constant value no longer dependent on the number of bays.

Coefficients k_1 through k_{20}, which determine the values of X_k^o and D as well as the expressions for M_ϕ on the noncontinuous beam, result from small differences of large figures which are sensitive to slight mistakes and which, therefore, require high accuracy and the use of calculating machines or other time-consuming methods. To avoid this, all expressions have been developed in power series* and it was found to be sufficiently accurate for the commonly encountered ϕ_k values to use only the first two members of each series. The first column in Table 1 gives the exact value, the second column the approximation, whereas the values in Tables 2 and 3 are based entirely on the approximate solution.

For those frames where ϕ_k exceeds 75 deg (which is a rather unusual case) the power series approach looses its accuracy, and it is recommended that one should use the exact values rather than those obtained from a power series. This can be done easily by substituting in Table 2 the exact values for k_4, k_5, etc., from Table 6 for the approximations from Table 3.

Here now is a brief description of the so-called shear loadings, values for which may be found on Line 6 of Table 1 and on Lines 4 and 8 of Table 2. This loading is due to the load transfer from the shell onto the supporting frame. It follows the shear distribution in the shell shown in Fig. 3, where

$$w_\phi = \frac{WQ_\phi}{I_s} \qquad (2)$$

*A table giving the power series for various trigonometric functions was published by the author. The values given therein were used in developing all formulas in this paper; Zweig, Alfred, Discussion of "Cylindrical Shell Analysis Simplified by Beam Method," ACI JOURNAL, *Proceedings* V. 55, No. 6, Part 2, Dec. 1959, p. 1596.

In this equation, w_ϕ represents the tangential shear per unit length in the shell at the frame support, W is the total shell load and Q_ϕ is the statical moment of the shell section in relation to the horizontal line at angle ϕ, and I_s is the moment of inertia of the shell section. Then

$$Q_\phi = 2r^2t\left(\sin\phi - \frac{\phi}{\phi_2}\sin\phi\right) \quad \text{.....(3)}$$

$$I_s = r^3t\left[\phi_2 + \sin\phi_2\left(\cos\phi_2 - 2\frac{\sin\phi_2}{\phi_2}\right)\right] \quad \text{.....(4)}$$

with t representing the shell thickness.

This assumption will be true to the same extent and within the same limitations as it is permissible to use the so-called beam method for the shell analysis as outlined in Chinn's paper.* It further presupposes that radial shears are negligible in comparison with the tangential shear which is the basis for every practical shell analysis.

Comparing the moments resulting from the so-called shear loading with those obtained from the surface loading, we find that the shear loading will give values 20 to 30 percent higher than those resulting from the commonly used surface loading. It, therefore, does not appear

*Chinn, James, "Cylindrical Shell Analysis Simplified by Beam Method," ACI JOURNAL, *Proceedings* V. 55, No. 11, May 1959, pp. 1183-1192.

TABLE 3 — k VALUES FOR VARIOUS ϕ VALUES

ϕ_2		$k_1 = \frac{30\phi_2 + \phi_2^3}{45}$	$k_2 = \frac{15\phi_2 - \phi_2^3}{45}$	$k_3 = \frac{\phi_2^3}{6} - \frac{7\phi_2^4}{180}$	$k_4 = \frac{4\phi_2^5}{15} - \frac{16\phi_2^7}{315}$	$k_5 = \frac{2}{15}\phi_2^4 - \frac{13}{315}\phi_2^6$
Degrees, min	Radians					
28-39	0.5	0.33611	0.16389	0.03924	0.00793	0.00769
34-23	0.6	0.40480	0.19520	0.05496	0.01932	0.01535
40-6	0.7	0.47429	0.22571	0.07233	0.04064	0.02715
45-50	0.8	0.54471	0.25529	0.09074	0.07673	0.04379
51-34	0.9	0.61620	0.28380	0.10948	0.13317	0.06555
57-18	1.0	0.68889	0.31111	0.12778	0.21588	0.09206

ϕ_2		$k_6 = \frac{\phi_2^3}{3} - \frac{\phi_2^5}{30}$	$k_7 = \frac{\phi_2^3}{6} - \frac{\phi_2^4}{20}$	$k_8 = \frac{\phi_2^3}{4} - \frac{\phi_2^4}{16}$	$k_9 = \frac{3}{14}\phi_2^3 - \frac{563}{8820}\phi_2^4$	$k_{10R} = \frac{7}{48}\phi_2^3 - \frac{13}{360}\phi_2^4$
Degrees. min	Radians					
28-39	0.5	0.04063	0.03854	0.05859	0.04972	0.03420
34-23	0.6	0.06941	0.05352	0.08190	0.06908	0.04782
40-6	0.7	0.10873	0.06967	0.10749	0.08995	0.06279
45-50	0.8	0.15975	0.08619	0.13440	0.11136	0.07854
51-34	0.9	0.22332	0.10220	0.16149	0.13215	0.09443
57-18	1.0	0.30000	0.11667	0.18750	0.15046	0.10972

TABLE 3 (CONT.) — k VALUES FOR VARIOUS ϕ VALUES

ϕ_2		$k_{10L} = \frac{3}{16}\phi_2^2 - \frac{1}{24}\phi_2^4$	$k_{11} = \frac{2}{15}\phi_2^4 - \frac{16}{315}\phi_2^6$	$k_{12} = \frac{5}{24}\phi_2^4 - \frac{49}{720}\phi_2^6$	$k_{13} = \frac{11}{63}\phi_2^4 - \frac{4828}{727651}\phi_2^6$
Degrees, min	Radians				
28-39	0.5	0.04428	0.00754	0.01196	0.01005
34-23	0.6	0.06210	0.01491	0.02382	0.01989
40-6	0.7	0.08188	0.02603	0.04201	0.03479
45-50	0.8	0.10293	0.04129	0.06749	0.05528
51-34	0.9	0.12454	0.06049	0.10052	0.08114
57-18	1.0	0.14583	0.08254	0.14027	0.11107

ϕ_2		$k_{14} = \frac{2\phi_2^4}{15} - \frac{13}{315}\phi_2^6$	$k_{15} = \frac{\phi_2}{2} - \frac{\phi_2^3}{12}$	$k_{16} = \frac{\phi_2}{2} - \frac{7}{60}\phi_2^3$	$k_{17} = \frac{9}{14}\phi_2 - \frac{437}{2940}\phi_2^3$
Degrees, min	Radians				
28-39	0.5	0.00770	0.23958	0.23542	0.30285
34-23	0.6	0.01536	0.28200	0.27480	0.35361
40-6	0.7	0.02716	0.32142	0.30998	0.39902
45-50	0.8	0.04380	0.35733	0.34027	0.43818
51-34	0.9	0.06554	0.38925	0.36495	0.47021
57-18	1.0	0.09208	0.41667	0.38333	0.49422

ϕ_2		$k_{18} = \frac{3}{4}\phi_2 - \frac{11}{80}\phi_2^3$	$k_{19} = \frac{105\phi_2 - 19\phi_2^3}{240}$	$k_{20} = \frac{45\phi_2 - 7\phi_2^5}{80}$	$\bar{k}_2 = \frac{\phi_2}{6} - \frac{7}{180}\phi_2^3$
Degrees, min	Radians				
28-39	0.5	0.35781	0.20885	0.27031	0.07848
34-23	0.6	0.42030	0.24540	0.31860	0.09160
40-6	0.7	0.47784	0.27910	0.36374	0.10333
45-50	0.8	0.52960	0.30947	0.40520	0.11343
51-34	0.9	0.57476	0.33604	0.44246	0.12165
57-18	1.0	0.61250	0.35833	0.47500	0.12778

ϕ_2		$\bar{k}_6 = \frac{\phi_2^2}{3} - \frac{\phi_2^4}{30}$	$\bar{k}_7 = \frac{\phi_2}{6} - \frac{\phi_2^3}{20}$	$\bar{k}_8 = \frac{\phi_2}{4} - \frac{\phi_2^3}{16}$	$\bar{k}_9 = \frac{3}{14}\phi_2 - \frac{563}{8820}\phi_2^3$
Degrees, min	Radians				
28-39	0.5	0.08126	0.07708	0.11718	0.09944
34-23	0.6	0.11568	0.08920	0.13650	0.11513
40-6	0.7	0.15533	0.09953	0.15356	0.12850
45-50	0.8	0.19969	0.10774	0.16800	0.13920
51-34	0.9	0.24813	0.11355	0.17943	0.14683
57-18	1.0	0.30000	0.11667	0.18750	0.15046

to be on the safe side to replace the shear loads with the surface loads, at least in those cases where the so-called beam analysis for the shell is applicable.

The shear loading approach has the further advantage that the force component for the statically determinate arched beam at any section ϕ can be found directly from the following equations:

$$F_v = \frac{W r^2 t}{I_x}\left[\frac{\phi}{2} - \frac{\sin 2\phi}{4} + \frac{\sin \phi_k}{\phi_k}(\phi \cos \phi - \sin \phi)\right] \quad (5)$$

$$F_h = \frac{W r^2 t}{I_x}\left[\frac{\sin \phi_k}{\phi_k}(\cos \phi + \phi \sin \phi - \cos \phi_k) - \frac{\sin^2 \phi_k + \sin^2 \phi}{2}\right] \quad (6)$$

F_v represents the vertical and F_h the horizontal force component at Section ϕ.

Considering the relation expressed in Eq. (4) and substituting $x\phi_k$ for ϕ, we can rewrite Eq. (5) and (6), developing each term into a power-series, in the following manner:*

$$F_v = \frac{W}{112}[140x^3 - 84x^5 + 13\phi_k^2(x^3 - 2x^5 + x^7)] \quad (5a)$$

$$F_h = \frac{15}{16} W\left[\frac{1 - 2x^2 + x^4}{\phi_k} - \frac{\phi_k}{210}(19 + 39x^2 - 135x^4 + 77x^6)\right] \quad (6a)$$

These equations can be further simplified considerably if we apply them to specific loactions. For the most commonly investigated locations, i.e., at the support, at the quarter point, and in the center, we obtain the following values:

At the support (i.e., for $\phi = \phi_k$ or $x = 1$)

$$F_v = \frac{W}{2} \quad (7a)$$

and

$$F_h = 0 \quad (7b)$$

At the quarter point (i.e., for $\phi = \phi_k/2$ or $x = 1/2$)

$$F_v = \frac{W}{128}\left(17 + \frac{117}{112}\phi_k^2\right) \quad (8a)$$

and

$$F_h = \frac{135}{256} W\left(\frac{1}{\phi_k} - \frac{51}{280}\phi_k\right) \quad (8b)$$

*For frames with an ϕ_k value larger than 75 deg it is not recommended using the approximation of Eq. (5a) and (6a), but rather to obtain the exact values form Eq. (5) and (6).

TABLE 4 — k AND A VALUES FOR VARIOUS n VALUES

n	$K_{k(n)} = 1 - \frac{\beta^{n-k} + \beta^k}{\beta^n + 1}$								$A_n = \frac{\beta^{n-1}}{\beta^{n+1}} \frac{\beta+1}{\beta-1}$
	ϕ_k \ k	0	1	2	3	4	5	6	
1	0.5	0	0						1.000
	0.7	0	0						1.000
	0.9	0	0						1.000
2	0.5	0	1.488	0					0.512
	0.7	0	1.477	0					0.523
	0.9	0	1.463	0					0.537
3	0.5	0	1.196	1.196	0				0.608
	0.7	0	1.192	1.192	0				0.615
	0.9	0	1.188	1.188	0				0.624
4	0.5	0	1.277	0.865	1.277	0			0.581
	0.7	0	1.269	0.872	1.269	0			0.590
	0.9	0	1.259	0.880	1.259	0			0.602
5	0.5	0	1.256	0.950	0.950	1.256	0		0.588
	0.7	0	1.250	0.952	0.952	1.250	0		0.597
	0.9	0	1.242	0.955	0.955	1.242	0		0.607
6	0.5	0	1.262	0.927	1.035	0.927	1.262	0	0.586
	0.7	0	1.255	0.931	1.033	0.931	1.255	0	0.595
	0.9	0	1.246	0.936	1.030	0.936	1.246	0	0.606

At the center of the span (i.e., for $\phi = 0$ or $x = 0$)

$$F_v = 0 \qquad (9a)$$

and

$$F_h = \frac{15}{16} W \left(\frac{1}{\phi_k} - \frac{19}{210} \phi_k \right) \qquad (9b)$$

Since these equations pertain to the statically determinate noncontinuous beam, final values will be obtained by adding to these values for F_h, the influence of the horizontal thrust, and for F_v the influence of the vertical reactions due to the moments $X_k{}^c$ over the supports.

If, on the other hand, surface loads are used in the design of the shell supporting frames, the force components at each section can be obtained only by integrating the normal shell forces in the tangential direction over the entire shell length, a rather cumbersome procedure which can be avoided with the previous direct solution.

The expressions for all values pertaining to the shear loading are rather involved and the power series approach becomes, therefore, especially advantageous when used for this loading type.

Summarizing the whole procedure, we find that the solution of the continuous arched frame resolves itself into three successive steps:

1. Establish moments over the support of the continuous arched beams with the help of Tables 2, 3, and 4

2. Determine horizontal deflections D_L and $D_{H=1}$, also with the help of Tables 2, 3, and 4, and, from them, the horizontal thrust H; add the elongation of the tie where necessary

3. Determine final moments by adding to the moments on the non-continuous arched beam taken from Table 1 the influence of the moments over the support and also the moments resulting from the horizontal thrust

This method will be demonstrated with a practical example.

EXAMPLE

Use of the design method and of the tables will be illustrated by calculating the supporting frame for Example 2 from ASCE *Manual* No. 31.*

The roof shells are assumed to extend over four bays and the supporting frame is assumed to have the proportions and properties indicated in Fig. 4. The dead weight of the frame itself is neglected to simplify the problem; but its influence can be easily included, if so desired. The analysis pertains to an exterior frame which supports one half of a shell span of 62 ft. The dead load and the uniformly distributed live load are assumed to act on the frame following the shear distribution shown in Fig. 3.

It is further assumed that the shell is acting in the center of gravity of the supporting beam, as shown in Section X-X in Fig. 4. Should the shell be attached, instead, either to the top or the bottom of the beam, the additional moments resulting therefrom must be considered separately. This does not present any difficulty if the force components at each section are known. How they are established is demonstrated further on.

Influence of dead load

Since the frame is symmetrical for dead load, we will analyze only the first two bays:

Moments over support of continuous arched beam

For dead load

$\phi_k = 40$ deg $= 40\pi/180 = 0.7$ $W = 31 \times 0.7 \times 2 \times 1457 = 63{,}200$ lb $\sin \phi_k = 0.6428$ $n = 4$ from Table 4 for $n = 4$ and $\phi_k = 0.7$ $K_{0(4)} = 0$, $K_{1(4)} = 1.269$ $K_{2(4)} = 0.872$; from Table 3 for $\phi_k = 0.7$ $\overline{K_2} = 0.1285$ and from Table 2 Line 4 $X_0^{(o)} = 0$ $X_1^{(o)} = -63.2 \times 31.0 \times 0.1285 \times 1.269 = -319$ ft-kips $X_2^{(o)} = -63.2 \times 31.0 \times 0.1285 \times 0.872 = -220$ ft-kips

Design of Cylindrical Concrete Shell Roofs, ASCE Manual No. 31, American Society of Civil Engineers, New York, 1956, pp. 58-63.

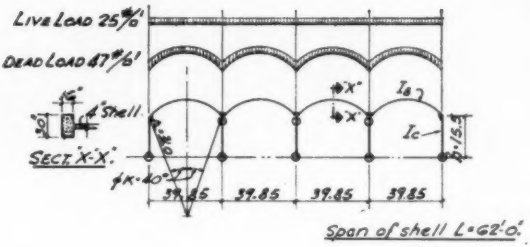


Fig. 4 — Frame shape and loading with interior columns not rigidly connected to arched frame

For horizontal force $H = 1$

For $\phi_k = 0.7$ from Table 3 $\overline{K_c} = 0.1553$; for $m = r/h = 31.0/15.5 = 2.0$ from Table 2, Line 1, $X_c^{(c)} = -15.5\,[1 - 0] = -15.5$ ft-kips; $X_1^{(c)} = -15.5\,[1 - (1 + 2 \times 0.1553)\ 1.269] = +10.28$ ft-kips; $X_2^{(c)} = -15.5\,[1 - (1 + 2 \times 0.1553)\ 0.872] = +2.22$ ft-kips.

Final moments over supports

The final moments over the support are obtained by adding to the moments of the continuous beam the influence of the horizontal thrust. The horizontal thrust, according to the following section, is 13.2 kips; therefore $X_1^{(c)} = -319 + 13.2 \times 10.28 = -183$ ft-kips and $X_2^{(c)} = -220 + 13.2 \times 2.22 = -192$ ft-kips.

Horizontal thrust *H*

$m = 2$, $a = I_b/I_c = 1$; for $\phi_k = 0.7$ from Table 3 $K_4 = 0.0406$, $K_2 = 0.0900$, $\overline{K_c} = 0.1553$, $K_0 = 0.1087$, $K_{12} = 0.0348$; from Table 4 for $n = 4$ $\phi_k = 0.7$ $A_n = 0.590$; therefore, from Table 2, Line 1, $S = (0.590/2) - 0.1553\,(4 - 0.590) = -0.234$ and $Z = \frac{2}{3} \times \frac{1}{8} + 2 \times 0.590/2 \times 0.7/2 + 0.1087 = 0.3539$. Now $E_s I_s D_{H=1} = r^3(4 \times 0.0406 - 2 \times 0.234 \times 0.1087 + 0.3539) = 0.4654\, r^3$ and from Table 2, Line 3 $E_s I_s D_{1=4} = -W_r{}^3\,(4 \times 0.0348 - 2 \times 0.234 \times 0.0900) = -0.0971\, W_r{}^3$; hence according to Eq. (1)

$$H = -W\,\frac{0.0971}{0.4654} = -0.208\,W = -13.2 \text{ kips}$$

Final moments

Moments on noncontinuous beams

Due to dead load according to Table 1, Line 6, at $\phi = \phi_k$ $M_\phi = 0$;

At $\phi = \frac{\phi_k}{2}$ and $\phi = \frac{-\phi_k}{2}$: $M_\phi = \frac{3}{16}\ 63.2 \times 31 \left(\frac{485}{384}\ 0.7\ \frac{-14455}{43008}\ 0.7^3 \right)$

$= 283$ ft-kips at $\phi = 0$ $M_\phi = \frac{63.2 \times 31}{4} \left(\frac{11}{8}\ 0.7\ \frac{-235}{672}\ 0.7^3 \right) = 411$ ft-kips

TABLE 5 — FINAL MOMENTS

Location of moments		Moments on noncontinuos beam, ft-kips, due to							Σ 1, 5, 6, 7 Final moments, ft-kips
		Dead load	$H = -1$	$X_1 = -1$	$X_2 = -1$	$H = -13.2$	$X_1 = -183$	$X_2 = -192$	
		1	2	3	4	5	6	7	
First bay	$\phi = 0$	0	−15.5	0		−204	0		−204
	$\phi = \frac{\phi_k}{2}$	+283	−17.25	−0.234		−228	− 43		+ 12
	$\phi = \phi_k$	+411	−15.05	−0.500		−199	− 91		+121
	$\phi = -\frac{\phi_k}{2}$	+283	− 9.03	−0.765		−119	−140		+ 24
	At support	0	0	−1.000	0	0	−183	0	−183
Second bay	$\phi = \frac{\phi_k}{2}$	+283	− 5.40	−0.765	−0.234	− 71	−140	− 45	+ 27
	$\phi = \phi_k$	+411	− 7.30	−0.500	−0.500	− 96	− 91	− 95	+129
	$\phi = -\frac{\phi_k}{2}$	+283	− 5.40	−0.234	−0.765	− 71	− 43	−146	+ 23
	$\phi = -\phi_k$	0	0	0	−1.000	0	0	−192	−192

Moments due to $X_{(1)} = -1$ *ft- kip*

According to Table 1, Lines 1 and 2

	1st Bay		2nd Bay
at $\phi = \phi_0$	$M_\phi = 0$	$= 0.00$	$M_\phi = -1.00$
$\phi = \frac{\phi_0}{2}$	$M_\phi = -\frac{1}{4}\left(1 - \frac{0.7^2}{8}\right)$	$= -0.234$	$M_\phi = -0.765$
$\phi = 0$	$M_\phi = -\frac{1}{2}$	$= -0.500$	$M_\phi = -0.500$
$\phi = -\frac{\phi_0}{2}$	$M_\phi = -\frac{3}{4}\left(1 + \frac{0.7^2}{24}\right)$	$= -0.765$	$M_\phi = -0.234$
$\phi = -\phi_0$	$M_\phi = -1$	$= -1.000$	$M_\phi = -0.000$

Moments due to $X_{(2)} = -1$ are the same as those for $X_{(1)} = -1$, except that the above values apply to the second and third bay instead of the first and second bay.

Due to $H = 1$ from Table 1, Line 3 for the second bay and from Line 3 together with Line 1 for the first bay.

1st bay

at $\phi = \phi_0$ $M_\phi = -15.5$ ft-kips

at $\phi = \frac{\phi_0}{2}$ $M_\phi = -15.5 \times \frac{3}{4}\left(1 + \frac{0.7^2}{24}\right) - \frac{3}{8} 31.0\left(0.7^2 - \frac{5}{48} 0.7^4\right)$

$= -11.85 - 5.40 = -17.25$ ft-kips

$\phi = 0$ $M_\phi = -15.5 \times \frac{1}{2} - \frac{1}{2} \times 31 \left(0.7^2 - \frac{0.7^4}{12}\right)$

$= -7.75 - 7.3 = -15.05$ ft-kips

at $\phi = -\frac{\phi_0}{2}$ $M_\phi = -15.5 \times \frac{1}{4}\left(1 - \frac{0.7^2}{8}\right) - \frac{3}{8} \times 31 \left(0.7^2 - \frac{5}{48} 0.7^4\right)$

$= -3.63 - 5.40 = -9.03$ ft-kips

2nd bay

at $\phi_0 = \phi_0$ and $\phi = -\phi_0$ $M_\phi = 0$ at $\phi = \phi_0/2$ and $\phi = -\phi_0/2$ $M_\phi = -5.40$ ft-kips at $\phi = 0$ $M_\phi = -7.30$ ft-kips

Final moments on continuous frame

The final moments on the continuous frame are obtained by adding to the moments on the noncontinuous beam the influence of the moments $X_1 = 183$ ft-kips, $X_2 = 192$ as well as the influence of the horizontal thrust of $H = 13.2$ kips. This is done in Table 5.

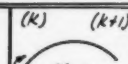
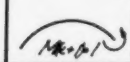
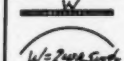
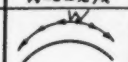
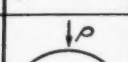
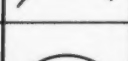
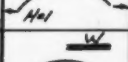
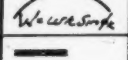
Force components

The force components on the noncontinuous beams are found with the help of Eq. (7), (8), and (9) as follows:

At $\phi = \phi_0$

$$F_0 = \frac{63.2}{2} = 31.6 \text{ kips}, \quad F_0 = 0$$

TABLE 6 — ANGULAR ROTATIONS AND HORIZONTAL DEFLECTIONS FOR VARIOUS LOADINGS

	LOADING	$EI\Theta_{k(k+1)} = \int M_{k(x)} M_{k(x)} \, ds/\rho$	$EI\Delta^{(k)}_{k(k+1)} = \int M_{k(x)} M_{k(x)} \, ds/\rho$
1		$\frac{k}{4\sin^2\phi_k}\left(\frac{2}{3}\phi_k \sin^2\phi_k + \phi_k - \sin\phi_k\cos\phi_k\right) = \frac{k}{45}\left(30\phi_k + \phi_k^3\right) = k K_1$	$k^2(\sin\phi_k - \phi_k\cos\phi_k) = \frac{k^2}{3}\left(\phi_k^2 - \frac{\phi_k^4}{10}\right) = k^2 K_6$
2		$\frac{k}{4\sin^2\phi_k}\left(2\phi_k\sin^2\phi_k - \phi_k + \sin\phi_k\cos\phi_k\right) = \frac{k}{45}\left(15\phi_k - \phi_k^3\right) = k K_2$	$k^2(\sin\phi_k - \phi_k\cos\phi_k) = \frac{k^2}{3}\left(\phi_k^2 - \frac{\phi_k^4}{10}\right) = k^2 K_6$
3		$\frac{Wk^2}{8}\left(\cos\phi_k + 2\phi_k\sin\phi_k - \frac{\phi_k}{\sin\phi_k}\right) = Wk^2\left(\frac{\phi_k^2}{6} - \frac{7}{180}\phi_k^4\right) = Wk^2 K_3$	$\frac{Wk^3}{4}\left(\phi_k\cot\phi_k + \frac{4}{3}\sin^2\phi_k - \cos^2\phi_k - 2\phi_k\sin\phi_k\cos\phi_k\right) = Wk^3\left[\frac{2}{15}\phi_k^4 - \frac{13}{315}\phi_k^6\right] = Wk^3 K_5$
4		$\frac{Wk^2}{2}\left(2\cos\phi_k + \phi_k\sin\phi_k - \frac{2\sin\phi_k}{\phi_k}\right) = Wk^2\left(\frac{\phi_k^2}{6} - \frac{\phi_k^4}{20}\right) = Wk^2 K_7$	$\frac{Wk^3}{2}\left(\frac{9\sin\phi_k\cos\phi_k}{2\phi_k} - \frac{7}{2}\cos^2\phi_k - 1 - 2\phi_k\sin\phi_k\cos\phi_k + \frac{5}{2}\sin^2\phi_k\right) = Wk^3\left[\frac{1}{15}\phi_k^4 - \frac{16}{315}\phi_k^6\right] = Wk^3 K_{11}$
5		$\frac{Wk^2d}{2I_0}\left(3\phi_k\cos^2\phi_k + \phi_k\cos\phi_k\sin\phi_k + \frac{5}{3}\phi_k^2\sin\phi_k - 3\sin\phi_k - \frac{3\sin^2\phi_k\cos\phi_k}{\phi_k}\right) = Wk^2\left(\frac{3}{14}\phi_k^2 - \frac{563}{3820}\phi_k^4\right) = Wk^2 K_9$	$\frac{Wk^3d}{I_0}\left[\frac{1}{\phi_k}\left(3\sin^2\phi_k\cos^2\phi_k + 2\sin^4\phi_k\right) + \frac{7}{2}\sin^2\phi_k\cos\phi_k - \phi_k\left(\frac{15}{4} + \sin^2\phi_k\cos^2\phi_k - 3\sin^2\phi_k\right) - \frac{5}{2}\sin\phi_k\cos\phi_k - \frac{5}{3}\phi_k^2\sin\phi_k\cos\phi_k\right] = Wk^3\left[\frac{1}{63}\phi_k^4 - \frac{4316}{72765}\phi_k^6\right] = Wk^3 K_{13}$
6		$\frac{Pk^2}{2}\left(\cos\phi_k + \phi_k\sin\phi_k - 1\right) = Pk^2\left(\frac{\phi_k^2}{4} - \frac{\phi_k^4}{16}\right) = Pk^2 K_8$	$Pk^3\left(\frac{1}{2}\sin^2\phi_k + \cos\phi_k - \phi_k\sin\phi_k\cos\phi_k - \cos^2\phi_k\right) = Pk^3\left(\frac{5}{24}\phi_k^4 - \frac{49}{720}\phi_k^6\right) = Pk^3 K_{12}$
7		$k^2(\sin\phi_k - \phi_k\cos\phi_k) = \frac{k^2}{3}\left(\phi_k^2 - \frac{\phi_k^5}{10}\right) = k^2 K_6$	$k^3\left(2\phi_k\cos^2\phi_k - 3\sin\phi_k\cos\phi_k\right) = k^3\left(\frac{4}{15}\phi_k^5 - \frac{16}{315}\phi_k^7\right) = k^3 K_4$
8		$\frac{Wk^2}{4\sin^2\phi_k}\left[\phi_k\sin^2\phi_k - \phi_k\sin\phi_k + \frac{2}{3}\sin^2\phi_k\cos\phi_k - \frac{5}{3}\cos\phi_k + \frac{5}{3}\right] = Wk^2\left[\frac{7}{48}\phi_k^2 - \frac{13}{360}\phi_k^4\right] = Wk^2 K_{10a}$	$\frac{Wk^3}{8}\left(\phi_k\cot\phi_k + \frac{4}{3}\sin^2\phi_k - \cos^2\phi_k - 2\phi_k\sin\phi_k\cos\phi_k\right) = Wk^3\left[\frac{2}{15}\phi_k^4 - \frac{13}{315}\phi_k^6\right] = Wk^3 K_{14}$
9		$\frac{Wk^2}{4\sin^2\phi_k}\left[\phi_k\sin^2\phi_k + \frac{1}{3}\sin^2\phi_k\cos\phi_k + \frac{2}{3}\cos\phi_k - \frac{5}{3}\right] = Wk^2\left[\frac{3}{16}\phi_k^2 - \frac{1}{24}\phi_k^4\right] = Wk^2 K_{10b}$	$\frac{Wk^3}{8}\left(\phi_k\cot\phi_k + \frac{4}{3}\sin^2\phi_k - \cos^2\phi_k - 2\phi_k\sin\phi_k\cos\phi_k\right) = Wk^3\left[\frac{2}{15}\phi_k^4 - \frac{13}{315}\phi_k^6\right] = Wk^3 K_{16}$

At $\phi = \frac{\phi_b}{2}$

$$F_v = \frac{63.2}{128}\left(17 + \frac{117}{112}0.7^3\right) = 8.66 \text{ kips}$$

$$F_h = \frac{135}{256}63.2\left(\frac{1}{0.7} - \frac{51}{280}0.7\right) = 43.60 \text{ kips}$$

At $\phi = 0$

$$F_v = 0 \quad \text{and} \quad F_h = \frac{15}{16}63.2\left(\frac{1}{0.7} - \frac{19}{210}0.7\right) = 81.50 \text{ kips}$$

In the first bay, these vertical force components must be modified by adding to them the vertical shear resulting from the moments over Supports 0 and 1, or

$$\Delta F_v = \frac{204 - 183}{39.85} = 0.55 \text{ kips}$$

and from the horizontal force components the horizontal thrust ($H = 13.2$) kips must be subtracted to obtain the final values as follows:

at $\phi = \phi_b$: $F_v = 31.6 + 0.55 = 32.15$ kips
$F_h = -13.2$ kips (compression)

at $\phi = \frac{\phi_b}{2}$: $F_v = 8.66 + 0.55 = 9.21$ kips
$F_h = 43.60 - 13.2 = 30.4$ kips (tension)

at $\phi = \phi_h$: $F_v = 0.55$ kips
$F_h = 81.50 - 13.2 = 68.3$ kips (tension)

The horizontal force components in the second bay will be identical with those in the first bay at corresponding locations. The vertical force components, in the second bay, however, will differ from those in the first bay, as the moments over Support 2 differ from those over Support 0.

Influence of a tie at column base

If we assume that the horizontal thrust cannot be resisted by the soil at the exterior footings, it becomes necessary to tie these footings together with a tie placed below the floor across the building. Under the assumption that this tie consists of two #9 reinforcing bars with a 2 sq in. cross-sectional area and a modulus of elasticity of 30,000,000 psi; and under the assumption further that the arched beam has a rectangular cross section of 16 x 30 in. with a moment of inertia of 36,000 in.^{4} and a modulus elasticity of 3,000,000 psi, the horizontal thrust can now be found in the following manner.

To the previously established value of $D_{H=1}$, we have to add the elongation of the tie, and since we used the deflection values multiplied by the

coefficient $E_B I_B$, we have to use the same multiplier for the tie elongation and we thus obtain

$$E_B I_B D_T = \frac{E_B I_B n l}{E_T A_T} = \frac{3{,}000{,}000}{30{,}000{,}000} \times \frac{36{,}000}{3.0 \times 144} 4 \times 39.85 = 1992$$

and

$$E_B I_B D_{H=1} = \left(0.2782 + \frac{1992}{29791} \right) r^3 = 0.5324\, r^3$$

From this follows the horizontal thrust

$$H = -\frac{0.0971}{0.5324} W = -0.182\, W = -11.5 \text{ kips}$$

It is interesting to note that through the introduction of the tie rod the horizontal thrust was reduced almost 13 percent. Consequently, the moments in the arched beam will be increased, but considerably more than 13 percent, as can be easily shown when recalculating the final moments in a similar manner to the method used for the frame without the tie rod.

TABLE 7 — U AND Q VALUES FOR LOADINGS

LOADING	U - VALUES		Q - VALUES
	ALL BAYS LOADED	ONLY ONE BAY LOADED	ALL BAYS LOADED
	$U = -\frac{2\Theta_K L}{\Theta_{(K-1)K}}$	$U = -\frac{\Theta_{KL}}{\Theta_{(K-1)K}}$	$Q = \frac{U}{2+S}$
H H	$\frac{r}{2}\left(-2\phi_K^2 + \frac{1}{15}\phi_K^4\right) =$ $-2 r k_{15}$	$\frac{r}{2}\left(-2\phi_K^2 + \frac{1}{15}\phi_K^4\right) =$ $-r k_{15}$	$\frac{r}{3}\left(-\phi_K^2 + \frac{1}{10}\phi_K^4\right) =$ $-r \bar{k}_6$
W	$Wr\left(-\phi_K + \frac{1}{6}\phi_K^3\right) =$ $-2 Wr k_{15}$	$\frac{Wr}{2}\left(-\phi_K + \frac{1}{6}\phi_K^3\right) =$ $-Wr k_{15}$	$Wr\left(-\frac{\phi_K}{6} + \frac{7}{180}\phi_K^3\right) =$ $-Wr \bar{k}_3$
W	$Wr\left(-\phi_K + \frac{7}{30}\phi_K^3\right) =$ $-2 Wr k_{16}$	$\frac{Wr}{2}\left(-\phi_K + \frac{7}{30}\phi_K^3\right) =$ $-Wr k_{16}$	$Wr\left(-\frac{\phi_K}{6} + \frac{1}{20}\phi_K^3\right) =$ $-Wr \bar{k}_7$
W	$Wr\left(-\frac{9}{7}\phi_K + \frac{437}{1470}\phi_K^3\right) =$ $-2 Wr k_{17}$	$\frac{Wr}{2}\left(-\frac{9}{7}\phi_K + \frac{437}{1470}\phi_K^3\right) =$ $-Wr k_{17}$	$Wr\left(-\frac{3}{14}\phi_K + \frac{563}{8820}\phi_K^3\right) =$ $-Wr \bar{k}_9$
P	$\frac{Pr}{2}\left(-3\phi_K + \frac{11}{20}\phi_K^3\right) =$ $-2 Pr k_{18}$	$\frac{Pr}{4}\left(-3\phi_K + \frac{11}{20}\phi_K^3\right) =$ $-Pr k_{18}$	$Pr\left(-\frac{1}{4}\phi_K + \frac{1}{16}\phi_K^3\right) =$ $-Pr \bar{k}_8$
W i i+1		$U_i = \frac{Wr}{240}(-105\phi_K + 19\phi_K^3) =$ $-Wr k_{19}$ $U_{i+1} = \frac{Wr}{80}(-45\phi_K + 7\phi_K^3) =$ $-Wr k_{20}$	

While these values are omitted here, the general remarks serve to emphasize the importance of properly evaluating the influence of the tie elongation.

Influence of live load

To find the influence of the live load, each bay will be separately loaded with a load of $W = L/2\ 1\ w = 31 \times 39.85 \times 25 = 30{,}900$ lb.

Moments over support of continuous beams

1st bay loaded—According to Table 2 for $n = 4$ and $i = 0$, and with the values for β from Table 8 we get for $p = 1$, 2, and 3, respectively

$$B^{0}_{1(0)} = \frac{\beta^7 - \beta}{\beta - 1}\ (\beta - 1) = 0.2536$$

$$B^{0}_{2(0)} = \frac{\beta^6 - \beta^2}{\beta - 1}\ (\beta - 1) = -0.064$$

$$B^{0}_{3(0)} = \frac{\beta^5 - \beta^3}{\beta - 1}\ (\beta - 1) = 0.0152$$

and, therefore, from Table 2, Line 8 with $K_{17} = 0.399$

$$X_1^{(0)} = -0.2536 \times 0.399 \times 30.9 \times 31 = -97.0 \text{ ft-kips}$$

$$X_2^{(0)} = \ \ 0.0641 \times 0.399 \times 30.9 \times 31 = \ \ 24.5 \text{ ft-kips}$$

$$X_3^{(0)} = -0.0152 \times 0.399 \times 30.9 \times 31 = \ -5.8 \text{ ft-kips}$$

2nd bay loaded—For $n = 4$, $i = 1$ and $g = 1$

$$B^{1}_{1(0)} = \frac{\beta^7 - \beta^3}{\beta - 1}\ (\beta - \beta^{-1}) = 0.190$$

and for $n = 4$, $i = 1$ and $p = 2$ and 3, respectively

$$B^{1}_{2(0)} = \frac{\beta^6 - \beta^2}{\beta - 1}\ (\beta^2 - \beta^{-1}) = 0.205$$

$$B^{1}_{3(0)} = \frac{\beta^5 - \beta^3}{\beta - 1}\ (\beta^2 - \beta^{-1}) = -0.049$$

and, therefore, again from Table 2, Line 8

$$X_1^{(0)} = -0.190 \times 0.399 \times 30.9 \times 31 = -72.5 \text{ ft-kips}$$

$$X_2^{(0)} = -0.205 \times 0.399 \times 30.9 \times 31 = -78.3 \text{ ft-kips}$$

$$X_3^{(0)} = \ \ 0.049 \times 0.399 \times 30.9 \times 31 = \ \ 18.7 \text{ ft-kips}$$

Due to the symmetry of the system, loading of the third and fourth bay will result in moments opposite those obtained from loading of the second and first bays, respectively. With this consideration in mind, we now find the moments over the supports for uniform live load in all bays as follows:

TABLE 8 — β VALUES FOR VARIOUS ϕ_k VALUES

ϕ_k	$S = 4 + 0.4\phi_k^2$	$\beta = -\frac{S}{2} + \sqrt{\frac{S^2}{4} - 1}$	β^2	β^3	β^4	β^5	β^6
0.5	4.100	−0.2605	0.0679	−0.0177	0.046	−0.0012	0.0003
0.7	4.196	−0.2537	0.0644	−0.0163	0.0041	−0.0011	0.0003
0.9	4.324	−0.2452	0.0601	−0.0147	0.0036	−0.0009	0.0002

TABLE 9 — VALUES OF X_0 IN FT-KIPS

a_s \ a_o	0	½	1	5	10	50	∞
0	−265	−237	−215	−123	−79	−21	0
½	−261	−234	−212	−122	−79	−21	0
1	−260	−234	−210	−121	−79	−21	0
5	−254	−228	−207	−120	−78	−21	0
10	−251	−226	−206	−119	−78	−21	0
50	−250	−224	−205	−119	−78	−21	0
∞	−249	−224	−204	−119	−78	−21	0

TABLE 10 — VALUES OF X_1 IN FT-KIPS

a_s \ a_o	0	½	1	5	10	50	∞
0	−180	−194	−206	−254	−275	−303	−315
½	−175	−190	−201	−250	−272	−304	−316
1	−172	−186	−198	−248	−272	−305	−318
5	−162	−177	−190	−244	−270	−305	−319
10	−158	−175	−188	−243	−270	−305	−319
50	−155	−171	−185	−242	−269	−305	−319
∞	−154	−170	−183	−240	−268	−305	−319

TABLE 11 — VALUES FOR X_2 IN FT-KIPS

a_s \ a_o	0	½	1	5	10	50	∞
0	−127	−137	−144	−178	−192	−212	−219
½	−138	−147	−154	−181	−195	−213	−219
1	−147	−154	−161	−186	−198	−214	−219
5	−170	−175	−179	−196	−205	−216	−219
10	−178	−182	−185	−200	−207	−216	−220
50	−187	−189	−191	−204	−209	−216	−220
∞	−189	−191	−192	−205	−210	−217	−220

$$X_1^{(e)} = -97 - 72.5 - 5.8 + 18.7 = -156.6 \text{ ft-kips}$$

$$X_2^{(e)} = 2\,(-78.3 + 24.5) = -107.6 \text{ ft-kips}$$

These values can be obtained also from the dead load moments by direct proportions. If all bays are loaded with 30.9 kips we get

$$X_1^{(e)} = -\frac{30.9}{63.2} \times 319 = -156.4 \text{ kips}$$

$$X_2^{(e)} = -\frac{30.9}{63.2} \times 220 = -107.5 \text{ kips}$$

which serves as a good check for the moments due to individual bay loadings.

Horizontal thrust *H*

With $E_B I_B\ D_{(H=1)}$ remaining the same as before, we have only to establish the $E_B I_B D_{Load}$ values according to Table 2, Line 8.

1st bay loaded—With $i = 0$ and $n = 4$, we get from Table 4 for $\phi_k = 0.7$

$K_{0(0)} = 0$, $K_{1(0)} = 1.269$; $T = 2/2 - (1/2 + 0.1553)\,(0 + 1.269) = 0.169$ (Table 2, Lines 6 and 8) and

$$E_B I_B D = -30.9 \times r^3 (0.03479 + 0.169 \times 0.09) = -1.545\, r^3$$

Therefore,

$$H = -\frac{1.545\, r^3}{0.465\, r^3} = -3.32 \text{ kips}$$

2nd bay loaded—With $i = 1$ and $n = 4$, we get from Table 4 for $\phi_k = 0.7$

$$K_{1(0)} = 1.269 \text{ and } K_{2(0)} = 0.872 \text{ and}$$

$$T = 2/2 - (1/2 + 0.1553)\,(1.269 + 0.872) = -0.402$$

Therefore,

$$E_B I_B D = -30.9\, r^3\,(0.03479 - 0.402 \times 0.09) = +0.0433\, r^3$$

and

$$H = +\frac{0.0433\, r^3}{0.278\, r^3} = 0.093 \text{ kips}$$

If all bays are uniformly loaded with 30.9 kips the horizontal thrust will be $2\,(3.32 - 0.093) = 6.454$ kips. This value can also be obtained from the dead load thrust by direct proportion,
or

$$H = \frac{30.9}{63.2}\,13.2 = 6.46 \text{ kips}$$

which serves also as good check for the figures involved. With the moments over the support and the horizontal thrust for live load

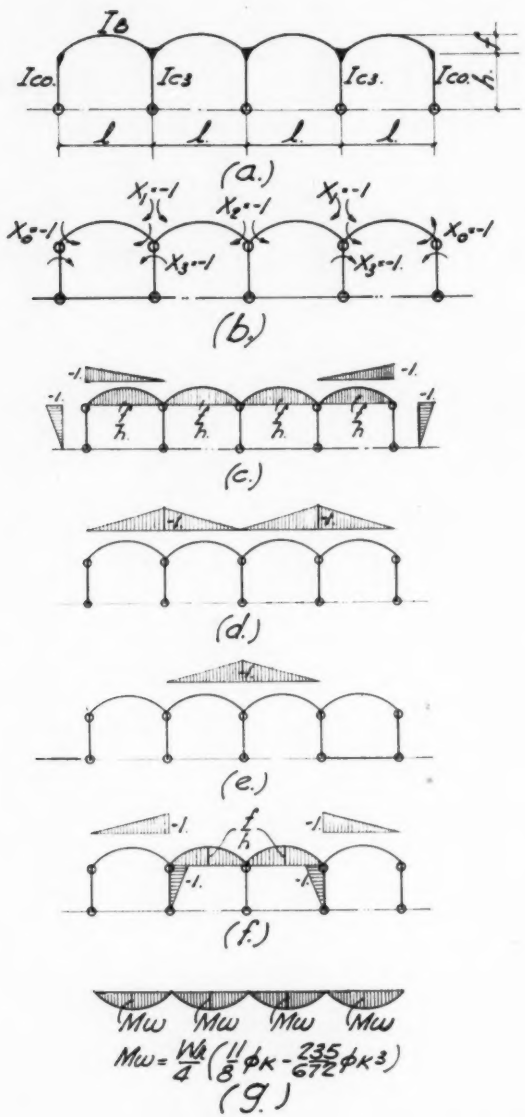


Fig. 4a-g — (a) Frame shape used in analysis of rigid connection between arched beams and columns; (b) Statically indeterminate unit moments; (c) Moments due $X_0 = -1$; (d) Moments due to $X_1 = -1$; (e) Moments due to $X_2 = -1$; (f) Moments due to $X_3 = -1$

known, the moments and force components at different sections can be found in the same way as shown for the dead load.

Influence of rigid connection between arched beams and interior columns

To evaluate the influence of a rigid connection between arched beams and interior columns, the system shown in Fig. 4a will be analyzed. It differs from the previously investigated system (shown in Fig. 4) only in the connection between interior columns and arched beams. The investigation will be confined to the action of the uniformly distributed dead load. To obtain a better insight, all moment values will be given as a function of the moment of inertia of the exterior and interior columns.

For the symmetrical dead load, this symmetrical system is statically indeterminate to the fourth degree. Moments X_0, X_1, X_2, and X_3 (see Fig. 4b) are chosen as the statically indeterminate values. They can be established with the help of the well-known equations of elasticity if the elastic deformations $\theta_{(i)k}$ (angular rotation of Section i due to a unit moment at Section k) and $\theta_{(i)L}$ (angular rotations of Section i due to load L) are known.

These values, observing the moment diagrams in Fig. 4c-4g, can be found with the help of Table 6 and the constants from Table 3.

With

$$\begin{aligned} k_1 &= 0.4743 & m &= r/h = 2.0 \\ k_2 &= 0.2257 & r &= 31.0 \\ k_4 &= 0.0406 & W &= 63.2 \text{ kip} \\ k_6 &= 0.1087 & a_0 &= I_b/I_{c0} \\ k_9 &= 0.0900 & & \\ k_{13} &= 0.0348 & a_3 &= I_b/I_{c3} \end{aligned}$$

We find

$$\frac{EI_B\,\theta_{00}}{2r} = k_1 + 2m^2 k_4 + 2m\,k_6 + \frac{a_1}{6} = 1.2339 + \frac{a_1}{6}$$

$$\frac{EI_B\,\theta_{11}}{2r} = 2k_1 = 0.9486$$

$$\frac{EI_B\,\theta_{22}}{2r} = k_1 = 0.4743$$

$$\frac{EI_B\,\theta_{33}}{2r} = k_1 + m^2 k_4 + \frac{a_3}{3m} = 0.6367 + \frac{a_3}{6}$$

$$\frac{EI_B\,\theta_{01}}{2r} = k_2 + 2m\,k_6 = 0.6605$$

$$\frac{EI_B\,\theta_{03}}{2r} = mk_6 = 0.2174$$

$$\frac{EI_B\,\theta_{03}}{2r} = k_2 + mk_6 - m^2 k_4 = 0.2807$$

$$\frac{EI_B\,\theta_{12}}{2r} = k_2 = 0.2257$$

$$\frac{EI_B\,\theta_{13}}{2r} = k_1 - m\,k_0 = 0.2569$$

$$\frac{EI_B\,\theta_{23}}{2r} = -m\,k_0 = -0.2174$$

$$\frac{EI_c\,\theta_{0L}}{2r} = -(2k_0 + 2mk_{12})\,Wr = -449.05 \text{ ft-kips}$$

$$\frac{EI_c\,\theta_{1L}}{2r} = -k_0 Wr = -352.66 \text{ ft-kips}$$

$$\frac{EI_c\,\theta_{2L}}{2r} = -k_0 Wr = -176.33 \text{ ft-kips}$$

$$\frac{EI_c\,\theta_{3L}}{2r} = (-k_0 + mk_{12})\,Wr = -39.97 \text{ ft-kips}$$

With these values, the equations of elasticity can be written as follows:

$$\left(1.2339 + \frac{a_0}{6}\right)X_0 + 0.6605\,X_1 + 0.2174\,X_2 + 0.2807\,X_3 = -449.05$$

$$0.6605\,X_0 + 0.9486\,X_1 + 0.2257\,X_2 + 0.2569\,X_3 = -352.66$$

$$0.2174\,X_0 + 0.2257\,X_1 + 0.4743\,X_2 - 0.2174\,X_3 = -176.33$$

$$0.2807\,X_0 + 0.2569\,X_1 - 0.2174\,X_2 + \left(0.6367 + \frac{a_3}{6}\right)X_3 = -39.97$$

Solving these simultaneous equations, we obtain

$$X_0 = -\frac{14919 + 6559\,a_3}{56.5 + 26.42\,a_3 + 13.21\,a_0 + 5.75\,a_0a_3} \quad \text{(10)}$$

$$X_1 = -\frac{10179 + 4167\,a_0 + 4071\,a_3 + 1837\,a_0a_3}{56.5 + 26.42\,a_3 + 13.21\,a_0 + 5.75\,a_0a_3} \quad \text{(11)}$$

$$X_2 = -\frac{7175 + 2903\,a_0 + 4994\,a_3 + 1264\,a_0a_3}{56.5 + 26.42\,a_3 + 13.21\,a_0 + 5.75\,a_0a_3} \quad \text{(12)}$$

$$X_3 = \frac{4687 - 196\,a_0}{56.5 + 26.42\,a_3 + 13.21\,a_4 + 5.75\,a_0a_3} \quad \text{(13)}$$

Values for X_0, X_1, X_2, and X_3 from Eq. (10) through (13) are tabulated in Tables 9 through 12 for various ratios a_0 and a_3.
The above tables confirm, first, some of the previously found values:

1. Assuming the moment of inertia of the interior and exterior columns to be zero, which corresponds to the case of the continuous beam without horizontal thrust, i.e., for $I_{c0} = I_{c3} = 0$ and $a_0 = a_3 = \infty$, we find $X_0 = X_3 = 0$ and $X_1 = -319$ ft-kips and $X_2 = -220$ ft-kips. These are the same values obtained with the help of Table 2.

2. Assuming the moment of inertia of the exterior columns to be equal to the moment of inertia of the arched beam, i.e., for $I_{c0} = I_B$

TABLE 12 — VALUES FOR X_3 IN FT-KIPS

a_3 \ a_0	0	½	1	5	10	50	∞
0	+83	+73	+65	+30	+15	−7	−15
½	+67	+59	+53	+25	+12	−6	−12
1	+57	+50	+44	+21	+10	−5	−10
5	+25	+22	+19	+ 9	+ 5	−2	− 5
10	+15	+13	+12	+ 5	+ 3	−1	− 3
50	+ 4	+ 3	+ 2	+ 1	+ 1	−1	− 1
∞	0	0	0	0	0	0	0

and $a_0 = 1$; and assuming further the moment of inertia of the interior columns to be zero or $a_3 = \infty$, we arrive at the system shown in Fig. 4, which was the basis of our main analysis. From Tables 9 to 12, we find for this system $X_0 = -204$ ft-kips, $X_1 = -183$ ft-kips, $X_2 = -192$ ft-kips and $X_3 = 0$. These are the same values we previously obtained from the final moments over supports. We can further draw from Tables 9 through 12 the following conclusions:

1. If we assume a rigid connection between interior columns and arched beams as shown in Fig. 4a and if we, first, assume the moment of inertia of exterior and interior columns to be the same and equal to the moment of inertia of the arched beam, i.e., $a_1 = a_3 = 1$, we find, from Tables 9 through 12, $X_0 = -210$ ft-kips, $X_1 = -198$ ft-kips, $X_2 = -161$ ft-kips, and $X_3 = +44$ ft-kips. This answer proves that our assumption of equal moments of inertia for interior and exterior columns was not justified since the moments at the interior column are only about one-fifth of those at the exterior columns, permitting a much smaller column section at the interior columns. This result will reflect the desire of most architects who prefer the smallest possible column sizes for interior columns.

2. If we now assume the interior columns to be of minimum size, designed to carry the vertical loads only, a 10 in. wide column with a cross-sectional area of 120 sq in. will suffice as compared to the 30-in. depth required for the large moments at the exterior columns. In this case $a_1 = 1$, $a_3 = 27$; and from Tables 9 through 12 by interpolation or directly from Eq. (10) through (13), we find $X_0 = -205$ ft-kips, $X_1 = -185$ ft-kips, $X_2 = -190$ ft-kips, and $X_3 = +5$ ft-kips.

These figures are of significance inasmuch as they justify our original assumption. Values for X_0, X_1, and X_2, obtained with complete disregard for the stiffness of a small interior column, differ less than 1 percent from the values we get if this stiffness is considered.

Under these conditions, it seems to be fully warranted to use the systems shown in Fig. 4 as the basis for our analysis, rather than the more accurate scheme shown in Fig. 4a, especially since the tables

presented in this paper enable us to arrive directly at the necessary moments and normal forces with sufficient accuracy, without the necessity of solving many simultaneous equations.

SUMMARY

Summarizing, we may state that the basic assumption of this paper (i.e., a nonrigid connection between interior columns and arched beams) reflects the structural behavior of the system, and the proposed method is justified in all those cases where interior columns are made of minimum sizes and designed for vertical loads only.

The moments in exterior columns and arched beams obtained in this way are sufficiently accurate and, while such an analysis does not furnish moments at the joint between interior columns and arched beams, these moments are small and a nominal reinforcement at these points will generally be satisfactory.

Where, on the other hand, contrary to the structural requirements interior columns are of sufficient rigidity to partake in the resistance of the horizontal thrust, the analysis based on the more simplified scheme shown in Fig. 1a should be replaced by the more accurate analysis based on the scheme shown in Fig. 4a as demonstrated previously under the heading "Influence of rigid connection between arched beams and interior columns."

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APPENDIX

MATHEMATICAL DERIVATIONS

Notation

a = I_b/I_c
g = numerical designation of a support to the left of loaded bay
h = column height
$i-(i+l)$ = numerical designation of support at both ends of loaded bay
k = numerical designation of any support
$k_1, k_2, k_3, \cdots,$ = coefficients defined and scheduled in Table 3
l = span of arched beam
m = r/h
n = number of bays of arched beam
p = numerical designation of support to the right of loaded bay $(i)-(i+1)$
q = $u/(2+S)$ function scheduled in Table 7 [see Eq. (A9)]
r = radius of shell and arched beam
s = $2\theta_{k(k)}/\theta_{(k-1)k}$ function scheduled in Table 8 [see Eq. (A3a)]

t = shell thickness
u = $-2\theta_{(k)L}/\theta_{(k-1)k}$ function scheduled in Table 7 [see Eq. (A3b)]
w = load per unit length
w_ϕ = shear load defined by Eq. (2)
x = ϕ/ϕ_k
$A_{(n)}$ = function defined and scheduled in Table 4 [see Eq. (A25)]
A_T = cross-sectional area of tie
$B^{(i)}{}_{g(n)}$, $B^{(i)}{}_{p(n)}$, $\bar{B}^{(i)}{}_{g(n)}$, $\bar{B}^{(i)}{}_{p(n)}$ = coefficients defined in Table 2
C_1, C_2 = constants for difference equations
D = lateral deflection at exterior column base
E_B = modulus elasticity of concrete pertaining to arched beams and exterior columns
E_T = modulus of elasticity of tie
F_v = vertical force component
F_h = horizontal force component
H = horizontal thrust
I_B = moment of inertia of arched beam
I_C = moment of inertia of exterior columns
I_s = moment of inertia of shell sections
$K_{k(n)}$ = function defined in Eq. (A12) and scheduled in Table 4
L = shell span
M_ϕ = moment for arched beam at Section ϕ
P = concentrated load
Q_ϕ = statical moment of shell section in relation to a horizontal line at Angle ϕ
S = constant defined in Table 2
T = constant defined in Table 2
W = total load
$X_k{}^e$ = moment over Support k
β = constant defined in Eq. (A7) and scheduled in Table 8
ϕ = angle locating a section of arched beam
ϕ_k = angle locating spring line of arched beam
$\theta_{(k-1)k}$ = angular rotation of Section $(k-1)$ due to a moment $X_k = 1$
$\theta_{(k)k}$ = angular rotation of Section (k) due to a moment $X_k = 1$
$\theta_{(k+1)k}$ = angular rotation of Section $(k+1)$ due to a moment $X_k = 1$
$\theta_{(k)L}$ = angular rotation of Section k due to a given load (scheduled in Table 6)
$\Delta^{(k)}{}_{(k-1)k}$ = horizontal deflection of beam $(k-1)-(k)$ due to a moment $X_k = 1$
$\Delta^{(k-1)}{}_{(k-1)k}$ = horizontal deflection of beam $(k-1)-(k)$ due to a moment $X_{(k-1)}$
$\Delta^{(L)}{}_{(k-1)k}$ = horizontal deflection of beam $(k-1)-(k)$ due to a given load (scheduled in Table 6)

Continuous beam analyzed by difference equations

Case 1 — All bays equally loaded. To analyze the continuous beam, we use the well-known three-moment theorem, which may be written in the following form:

$$\theta_{(k-1)k}\,X^e{}_{(k-1)} + 2\theta_{(k)k}\,X_k{}^e + \theta_{(k+1)k}\,X^e{}_{(k+1)} = 2\theta_{(k)L} \quad \text{(A1)}$$

$X^e{}_{(k-1)}$, $X^e{}_{(k)}$ and $X^e{}_{(k+1)}$ are the moments of the continuous beam at Supports $(k-1)$, (k), and $(k+1)$, respectively, whereas $\theta_{(k-1)k}$, $\theta_{(k)k}$, and $\theta_{(k+1)k}$ represent in this equation the rotations of the sections adjacent to Supports $(k-1)$, (k), and $(k+1)$, respectively, due to a pair of unit moments $X_k = 1$, one of each applied to either side of Section (k) on the noncontinuous beams $(k-1)(k)$ and $(k)(k+1)$. $\theta_{(k)L}$ represents the relative rotation of the two sections adjacent to Support (k) due to the given loading L applied to the noncontinuous beams $(k-1)(k)$ and $(k)(k+1)$ (see Fig. A1a and A1b).

According to the principle of virtual work for beams with constant moments of inertia:

$$\theta_{(k-1)k} = \frac{1}{E_b I_b} \int M^x_{(k-1)} M^x_{(k)} r d\phi \quad \text{(A1a)}$$

$$\theta_{(k)k} = \frac{1}{E_b I_b} \int M^x_{(k)} M^x_{(k)} r d\phi \quad \text{(A1b)}$$

$$\theta_{(k+1)k} = \frac{1}{E_b I_b} \int M^x_{(k+1)} M^x_{(k)} r d\phi \quad \text{(A1c)}$$

$$\theta_{(k)L} = \frac{1}{E_b I_b} \int M^x_{(L)} M^x_{(k)} r d\phi \quad \text{(A1d)}$$

where $M^x_{(k-1)}$, $M^x_{(k)}$, and $M^x_{(k+1)}$ represent the moments on the noncontinuous beams $(k-1)-(k)$ and $(k)-(k+1)$ due to unit moments $X_{(k-1)}$, X_k and $X_{(k+1)}$, respectively, and $M_L{}^x$ represents the moment on the noncontinuous beam due to the given loading.

For arched beams of the type discussed in this paper, the results of the integration for various moment combinations called for in Eq. (A1a)-(A1d) have been tabulated in the first column of Table 6, omitting here the rather involved derivations. In addition to the exact values, those obtained by using the power series approach are also given. Numerical values for coefficients k_1 through k_{10} are shown in Table 3.

Dividing Eq. (A1) by $\theta_{(k-1)k}$ and considering that $\theta_{(k-1)k} = \theta_{(k+1)k}$ we may then write the three-moment equation as follows:

$$X^c_{(k-1)} + s X^c_{(k)} + X^c_{(k+1)} = u \quad \text{(A2)}$$

wherein

$$s = \frac{2\theta_{(k)k}}{\theta_{(k-1)k}} \quad \text{(A3a)}$$

and

$$u = \frac{2\theta_{(k)L}}{\theta_{(k-1)k}} \quad \text{(A3b)}$$

Coefficient s varies only with the angle ϕ_k. Values for u depend on the loading and the beam shape. These values are tabulated in Table 7.

Eq. (A2) can be interpreted as a difference equation of the second order with constant coefficients for a given frame shape and loading.*

For the case under consideration (i.e., for all bays equally loaded) the right side of Eq. (A2), being independent of n, is constant.

The solution for the homogeneous difference equation

$$X^c_{(k-1)} + s X^c_{(k)} + X^c_{(k+1)} = 0 \quad \text{(A4)}$$

is

$$\bar{X}^c_k = C_1 \beta^{-k} + C_2 \beta^{-k} \quad \text{(A5)}$$

*The derivations in this paper are generally based on F. Bleich and E. Melan's book, *Die gewoehnlichen und partiellen Differenzenglechungen in der Baustatik, Julius Springer, 1927.*

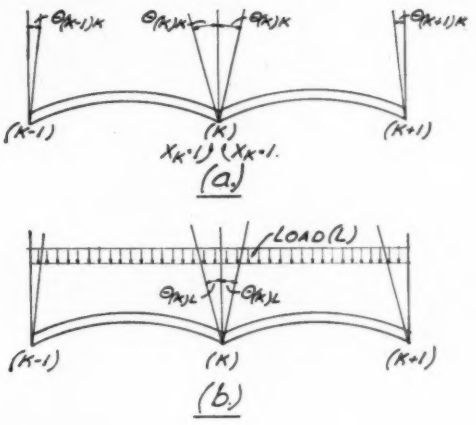

Fig. A1—Arched beam deformations: (a) Due to unit moment; (b) due to Load *L*

where β is a root of the characteristic equation

$$\beta^2 + s\beta + 1 = 0 \quad \text{(A6)}$$

or

$$\beta = -\frac{s}{2} + \sqrt{\frac{s^2}{4} - 1} \quad \text{(A7)}$$

Table 8 gives the values for s as well as for β and its powers for various angles ϕ_k.

The complete solution of the difference Eq. (A2) will be found if we can add to Eq. (A5) for the homogeneous Eq. (A4) a particular solution satisfying the nonhomogeneous Eq. (A2).

Since the right side of Eq. (A2) is a constant, the particular solution must also be a constant, the value of which is given by

$$\bar{\bar{X}}^o_k = \frac{u}{1 + s + 1} = \frac{u}{2 + s} \quad \text{(A8)}$$

Adding to the solution for homogeneous Eq. (A4) the particular solution [Eq. (A8)] and substituting in Eq. (A8)

$$q = \frac{u}{2 + s} \quad \text{(A9)}$$

we arrive at the following final and complete solution for the difference Eq. (A2)

$$X^o_{(k)} = \bar{X}^o_{(k)} + \bar{\bar{X}}^o_{(k)} = C_1\beta^k + C_2\beta^{-k} + q \quad \text{(A10)}$$

where C_1 and C_2 are constants to be determined by the boundary conditions:

$$X_0^o = 0 \quad \text{and} \quad X_n^o = 0$$

leading to the two equations

$$X_0^o = 0 = C_1 + C_2 + q \quad \text{and} \quad X_n^o = 0 = C_1\beta^n + C_2\beta^{-n} + q$$

from which it follows that

$$C_1 = -q \frac{1}{\beta^n + 1} \quad \text{(A11a)}$$

$$C_2 = -q \frac{\beta^n}{\beta^n + 1} \quad \text{(A11b)}$$

Substituting in Eq. (A10) the values for C_1 and C_2 [Eq. (A11a) and (A11b)] and with the notations

$$K_{k(n)} = 1 - \frac{\beta^{n-k} + \beta^k}{\beta^n + 1} \quad \text{(A12)}$$

we obtain the final and general equation for the moment X_k^e over the support k in the form

$$X_k^e = q\, K_{k(n)} \quad \text{(A13)}$$

In this equation $K_{k(n)}$ depends only on the shape and number (n) of bays, whereas q represents a load factor.

Table 7 gives the q values for different loadings. Table 4 tabulates the values of $K_{k(n)}$ for beams of one to six bays.

Case 2 — Each end of the continuous beam loaded with a pair of symmetrical or antisymmetrical moments (M). Considering first the case of the symmetrical moment pair shown in Fig. A2a, we shall see that the solution for this loading condition is considerably simpler than that of Case 1, wherein all bays were assumed to be loaded equally.

In the present case, with only the end supports loaded with a moment M, u in Eq. (A2) is zero, and our problem reduces to the homogeneous part [Eq. (A4)], for which the general solution is given in Eq. (A5).

The constants C_1 and C_2 are determined by the boundary conditions:

$$X_0^e = M \quad \text{and} \quad X_n^e = M$$

from which

$$C_1 = M \frac{1}{1 + \beta^n} \quad \text{(A11c)}$$

and

$$C_2 = M \frac{\beta^n}{1 + \beta^n} \quad \text{(A11d)}$$

Substituting in Eq. (A5) the values for C_1 and C_2 from Eq. (A11c) and (A11d) we find for a symmetrical moment pair

$$X_k^e = M(1 - K_{k(n)}) \quad \text{(A14a)}$$

wherein $K_{k(n)}$ is defined by Eq. (A12).

Turning now to the case of the antisymmetrical moment pair shown in Fig. A2b, the same consideration applies here as for the symmetrical moment pair, except that the constants C_1 and C_2 are determined by the revised boundary conditions:

$$X_0^e = -M \quad \text{and} \quad X_n^e = +M$$

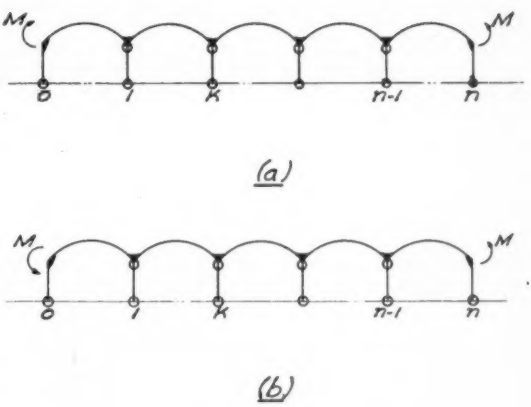

Fig. A2 — (a) Symmetrical Moment Pair *M*; (b) Antisymmetrical Moment Pair *M*

from which follows that

$$C_1 = -M \frac{1}{1-\beta^n} \qquad \text{(A11e)}$$

and

$$C_2 = M \frac{1}{1-\beta^n} \qquad \text{(A11f)}$$

consequently, for an antisymmetrical moment pair,

$$X_k^{\circ} = M \bar{K}_{k(n)} \qquad \text{(A14b)}$$

where

$$\bar{K}_{k(n)} = \frac{\beta^{n-k} - \beta^k}{1-\beta^n} \qquad \text{(A12a)}$$

Eq. (A14a) and (A14b) are important for the determination of moments due to wind loadings on the exterior columns. By expressing such an unsymmetrical loading as the sum of its symmetrical and antisymmetrical component, it is possible to apply the solution by difference equations, even to this loading case.

Case 3 — Only one bay loaded. To develop general formulas for the case of unsymmetrical loading of a single bay becomes rather cumbersome and, for this reason, the derivation of such formulas has been omitted here although it is possible to derive them and to develop even lines of influences with this method. However, the end result for one such loading condition (one-half bay loaded with a uniformly distributed load) has been scheduled on Line 10 of Table 2, as this loading may lead under certain circumstances to maximum moments.

If the loading is symmetrical about the center line of Span $(i)-(i+1)$, the formulas may be considerably simplified, especially if in the expression $\beta^{2n}-1$ we disregard β^{2n} as negligible with respect to 1, which is permissible for any value of n.

In this case

$$X_s^o = uB^i_{g(n)} \quad \text{(A15)}$$

and

$$X_p^o = uB^i_{p(n)} \quad \text{(A16)}$$

where

$$B^i_{g(n)} = \frac{\beta^{n-i} - \beta^{i+1}}{\beta - 1}(\beta^g - \beta^{-g}) \quad \text{(A17)}$$

and

$$B^i_{p(n)} = \frac{\beta^{n-p} - \beta^p}{\beta - 1}(\beta^{i+1} - \beta^{-i}) \quad \text{(A18)}$$

In these equations, g denotes a support to the left and p a support to the right of the loaded bay $i-(i+1)$.

It should be noted that for beams with five and more bays, the moment coefficients B for the supports of the loaded interior bay $i-(i+1)$ converge to the simple expression

$$B^i_{i(n)} = B^i_{(i+1)n} = \frac{\beta}{\beta - 1} \quad \text{(A17a)}$$

provided that

$$2 \leq i \leq n-3 \quad \text{and} \quad n \geq 5$$

Having thus developed the basic equations, it becomes possible to compute the moments over the support of a continuous arched beam for all loading conditions commonly encountered. This was done, and a tabulation of the results may be found in Table 2.

Horizontal deflection at base of outside column of continuous bent beam

Having analyzed in the previous section the continuous arched beam, we shall now develop the method used to determine the horizontal deflection D at the column base for any given loading.

This deflection is the sum total of the horizontal deflections of each arched beam $(k-1)-k$ for $k=1$ to $k=n$, plus the horizontal displacement of the column top in relation to the column base Δ_{col} for the two exterior columns

$$D = \Sigma \Delta^L_{(k-1)k} + 2\,\Delta_{col} \quad \text{(A19)}$$

Deflection $\Delta^L_{(k-1)k}$ due to a given loading can be found according to the principle of virtual work with the help of the following equation:

$$\Delta^L_{(k-1)k} = \frac{1}{E_B I_B} \int M^o_L M_{(H=1)}\, r d\phi \quad \text{(A20)}$$

where M_L^o represents the moment due to the given loading on the continuous beam, and $M_{(H=1)}$ the moment due to a horizontal force equal to 1 applied at the base of the arched beam (see Fig. 2).

The moment M_L^o on the continuous beam due to the given loading is nothing else but the sum of the moments on the noncontinuous beam due to this loading plus the influence of the moments $X_{(k-1)}$ and X_k over Supports $(k-1)$ and k, respectively. We may, therefore, write:

$$\Delta_{(k-1)k} = \Delta^L_{(k-1)k} + X_{(k-1)}\,\Delta^{(k-1)}_{(k-1)k} + X_{(k)}\,\Delta^{(k)}_{(k-1)k} \quad \text{(A21)}$$

where

$$\Delta^{L}_{(k-1)k} = \int M^{(N)}_{L}\, M^{(N)}_{(H=1)}\, r d\phi\, \frac{1}{E_s I_s} \qquad \text{(A22a)}$$

$$\Delta^{(k-1)}_{(k-1)k} = \int M^{(N)}_{(H_{k-1}=1)}\, M^{(N)}_{(H=1)}\, r d\phi\, \frac{1}{E_s I_s} \qquad \text{(A22b)}$$

$$\Delta^{k}_{(k-1)k} = \int M^{(N)}_{(H_{k}=1)}\, M^{(N)}_{(H=1)}\, r d\phi\, \frac{1}{E_s I_s} \qquad \text{(A22c)}$$

In these equations the index N indicates that all moments pertain to the noncontinuous beam and the subscript indicates the cause of the moment, such as $X_k = 1$, $H = 1$ or load L.

Omitting the tedious numerical work involved in the integration called for by Eq. (A22) the results may be found in Column 2 of Table 2. With the help of coefficients k_4 through k_8, and k_{11} through k_{14}, tabulated in Table 3, it is now possible to establish the horizontal deflection of each arched beam for a given loading.

The second term in Eq. (A19) representing the horizontal displacement of the column top in relation to the column base, consists of two elements (see Fig. 2): Δ^{r}_{col} accounts for the rotation of the column top and Δ^{d}_{col} accounts for the deflection of the column. The first part exists in all cases, the second only if the load is applied to the exterior column.

In our consideration this is the case, for instance, when a unit load is applied to the column base. For this loading

$$\Delta^{d}_{col} = \frac{h^3}{3\,EI_c} \qquad \text{(A23)}$$

where h represents the column height and I_c the moment of inertia of the columns.

Having thus obtained the horizontal deflection for each individual member according to Eq. (A21) and for the columns, where such a deflection exists, the next step required is the summation of these deflections for all n beams and, where necessary, also for the two columns.

In performing this summation, it becomes necessary to add the function $K_{k(n)}$ from the value $k = 1$ to $k = n$. According to the definition of Eq. (A12), $K_{k(n)} = 1 - (\beta^{n-k} + \beta^{k})/(\beta^{n} + 1)$. Using the laws of the geometrical progression, we obtain the following relationship:

$$\sum_{k=1}^{k=n} K_{k(n)} = n - A_{(n)} \qquad \text{(A24)}$$

where

$$A_{(n)} = \frac{\beta^{n-1}}{\beta^{n+1}}\,\frac{\beta + 1}{\beta - 1} \qquad \text{(A25)}$$

and similarly

$$\sum_{k=1}^{k=n} K_{(k-1)n} = n - A_{(n)} \qquad \text{(A26)}$$

It should be further observed that in the expressions for the deflections $\Delta^{k-1}_{(k-1)k}$ and $\Delta^{k}_{(k-1)k}$ in Eq. (A22b) and (A22c), $M^{N}_{(H=1)}$ is identical for all bays and equal to $r(\cos\phi - \cos\phi_k)$ except for the first and last bays where the influence of the moment at the exterior column top must be added. This additive moment for the two exterior bays equals the value of

$$h\,\frac{\sin\phi_k + \sin\phi}{2\,\sin\phi_k} \qquad \text{(A27)}$$

With these considerations in mind, the method will be demonstrated by establishing the D value for the case of a uniformly distributed load over all bays. In this case we have:

$$\sum_{k=1}^{k=n} \Delta^{(L)}_{(k-1)k} = n\,W r^3 k_c^*$$

$$\sum_{k=1}^{k=n} X_{(k-1)}\,\Delta^{(k-1)}_{(k-1)k} = -(r^2 k_c)\,W r \bar{k}_s\,(n - A_{(n)})^\dagger$$

$$\sum_{k=1}^{k=n} X_{(k)}\,\Delta^{(k)}_{(k-1)k} = -(r^2 k_c)\,W r \bar{k}_s\,(n - A_{(n)})^\ddagger$$

$$\Delta^{(h)}_{col} = 2\,W r^2 k_c h - 2\,W r \bar{k}_s K_{1(n)}\,r k_c h^\S$$

The last term $\Delta^{(r)}_{col}$ accounts for the moment integration due to the influence of the moment h at the exterior column top from the load $H = 1$. This integration extends only over the first and last bay.

Adding these four values and observing that

$$1 - k_s \frac{\bar{k}_s}{k_s} K_{1(n)} = A_{(n)} \qquad \text{(A28)}$$

and

$$k_c \bar{k}_s = \bar{k}_c k_s \qquad \text{(A29)}$$

we obtain

$$D = W r^3 (n\,k_c + 2\,k_s S) \qquad \text{(A30)}$$

where

$$S = \frac{A_{(n)}}{m} - \bar{k}_c\,(n - A_{(n)}) \qquad \text{(A31)}$$

Similar consideration led to the establishment of the deflection values D for other loading conditions as tabulated in Table 2.

All these values were checked with the use of the following theorem:

> A deformation on a statically indeterminate structure can be found by integrating the product of the moment, M_L, due to the given load, and the moment, M_1, due to a unit force applied in the direction of the unknown deformation. This integration is to extend over all members of the system and the result is to be divided by the product of the moment of inertia and modulus of elasticity, or

$$\Delta = \Sigma \int \frac{M_1 M_L}{EI}\,ds \qquad \text{(A32)}$$

This integration can be performed in three different ways:

1. Loads and unit force may be applied to the statically indeterminate system and both moments used in Eq. (A32) will then pertain to the statically indeterminate system.

2. The unit force may be applied to the statically indeterminate and the loads to any statically determinate system, in which case the M_1 values in Eq. (A32)

*Table 6, Line 3
†Table 2, Line 2 and Table 6, Lines 1 or 2
‡Table 2, Line 2 and Table 6, Lines 1 or 2
§Table 2, Line 2 and Table 6, Lines 2 and 3

pertain to the statically indeterminate and the M_L values to a statically determinate system.

3. The unit force may be applied to any statically determinate and the loads to the indeterminate system. Consequently, the M_1 values in Eq. (A32) pertain to the statically determinate and the M_L values to the statically indeterminate system.

This theorem, offering three different methods to obtain one and the same result, is especially helpful for checking purposes.

In this derivation leading to the D value for a uniformly distributed load, Eq. (A30), the third method was used since it lends itself best to a geometric interpretation of the procedure.

The second method, however, with the unit force acting on the statically indeterminate and the load on the determinate system, results in considerably simpler mathematical computations; and it was used in all instances to check the results obtained with the third method.

Such a check was especially valuable since it verified not only the correctness of all D values and, hence, of the formulas for the horizontal thrust but it also provided a proof for the correctness of the moments X_k^e over all supports.

Another check for the formulas derived in this paper was obtained by applying them to the case of a bent with straight, instead of arched beams. The simplified case can be easily analyzed by the Hardy Cross, or any other, method so that it was possible to check the validity of the formulas for this special condition, in which $\phi_k = 0$. To use the formulas for this case, Span $1 = 2r \sin \phi_k$ must be assumed as constant or the value r must be replaced by $1/2 \sin \phi_k$. With proper reductions the formulas will then lead to the same results as those arrived at by any other method.

It might be mentioned in conclusion that without such checks it would be almost impossible to present formulas like those developed in this paper with any assurance of accuracy.

Diseño de Marcos Arqueados Continuos para Soportar Cascaras Cylindricas

Presentación de un método para diseñar marcos arqueados contínuos, ofreciéndose tablas para usarse con las cargas más corrientes, así como distintas formas de marcos. El método persentado permite diseñar sin gran esfuerzo, estos marcos, los cuales normalmente requieren la solución simultánea de un número de ecuaciones. El apéndice ofrece las deducciones matemáticas de las fórmulas ofrecidas.

Calcul du Cadre Continu Arché, Portant Voiles Cylindriques

Une méthode se présente pour calcul des cadres continument archés et on offre une table pour user avec les charges plus ordinairement employées et les formes de cadres. La méthode permettra le calcul de ces cadres, lequel normalement exige la solution simultanée de quelques équations sans un grand effort. Un appendice pourvoit la dérivation mathématique des formules offertes.

Entwurf des durchgehend gewoelbten Rahmens zur Unterstuetzung zylindrischer Schalen

Es wird eine Methode fuer das Entwerfen durchgehend gebogener Rahmen dargestellt und Tabellen zur Benutzung mit den gebraeuchlichsten Belastungen und Rahmenformen geboten. Die Methode will den Entwurf dieser Rahmen, der normalerweise die gleichzeitige Loesung einer Anzahl von Gleichungen erfordert, ohne grosse Anstrengungen ermoeglichen. Ein Anhang liefert die mathematische Ableitung der gebotenen Formeln.

Title No. 58-23

Estimation of Heat of Hydration of Portland Cement

By KEN YONG and KUNG JEN-HSIA

Two methods of estimation of 7- and 28-day heats of hydration of portland cement from analytical data are studied by statistical analysis. In Method I, the heats of hydration are correlated with the calculated compound content, i.e., C_3S, C_2S, C_3A, and C_4AF as the independent variables, and in Method II, they are correlated with the corrected C_3S and C_2S and glass content as calculated by the series of equations derived by Dahl, as the independent variables. Comparisons with observed values show that predicted values calculated by Method II compare favorably with those by Method I, so that estimations can be more accurately and reliably made.

■ IN DESIGN AND CONTROL OF CEMENT compositions to meet heat of hydration specifications, the ability to estimate the heat of hydration at given ages from the compound compositions of the cement, such as C_3S, C_2S, C_3A, C_4AF, is essential. That the heat of hydration of cement at a certain age is significantly correlated with the amount of the major compounds in it, seems to have been established.[1] The problem, then, reduces to the evaluation of the contribution to the heat of hydration, at given ages by each percent of major compounds contained in the cement. Woods, Steinour, and Starke[1], in 1932, determined the heat evolved during hardening of a number of cements at various ages of hydration. The amount of heat evolution due to each of the four major compounds in the cement was calculated. Data at various ages of hydration have also been reported by Verbeck and Foster[2]. The contribution to the heat of hydration by each of the four major compounds has been calculated by the method of least squares.

Although the data reported in these two investigations were based on cements ranging widely in their chemical composition, they do not seem, however, to be generally applicable. For practical purposes, it is best for each plant to derive from its own data the necessary multiple correlation. This is true because: 1. Minor compounds in the cement

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contribute to the heat of hydration at a given age, however unimportant it may be. For a plant using raw materials of reasonably uniform composition the content of the minor compounds in cement may be expected to be comparatively constant. Under such a situation, the influence of the minor compounds can be taken into consideration by a constant in a correlation equation. 2. There is a problem of glass content of the cement. It has been reported by Lerch[3] that the glass content in the clinker, and hence the cement, has been found to remarkably influence the heat of hydration. Therefore, C_3A and C_4AF (which are present in the liquid above 1338 C) as calculated by the Bogue method can not be taken as such, but should be considered to solidify into glass. 3. If the clinkering temperature is assumed to be about 1400 C, which is normally the case, and the equilibrium mixture is to be frozen from the clinkering temperature, an approximate correction of C_3S and C_2S content, calculated by the Bogue method, may be applied.[5] In recent years, with the use of various types of air quenching coolers where forced draft instead of natural draft is generally used, these assumptions are justifiable. The correction for C_3S and C_2S and the calculation of the glass content of a cement may be made in accordance with the general classification key and the series of equations as derived by Dahl.[4] The multiple correlation equation at a given age may then be derived with corrected C_3S and C_2S and glass content as the independent variables.

The purpose of this paper is to present the results of hydration heat estimates at 7 and 28 days by: (1) using a correlation with C_3S, C_2S, C_3A, and C_4AF, as calculated from analytical data by the Bogue method, as the independent variables, Method I; (2) using a correlation equation with corrected C_3S and C_2S and glass content as the independent variables, Method II. The predicted values calculated by Methods I and II are compared with observed values to ascertain which of these methods will give better estimations. Also, the confidence range of the predicted values at significant probability levels are calculated for both methods to ascertain which of them will give smaller ranges. The effect of specific surface of cement, in sq cm per g, as determined by the Blaine permeability apparatus, is also discussed.

RESULTS

The heat of hydration was determined by the heat of solution method according to ASTM C 186-55; the chemical analysis according to ASTM C 114-53; and the specific surface according to ASTM C 204-55. In the calculation of C_3S by the Bogue method from the analytical data, the free CaO is deducted from the total CaO. The results are given in Table I.

The corrected C_3S and C_2S and glass content for the same cements calculated with the appropriate equations derived by Dahl, are given in Table II.

CALCULATION

Notation

$H_7(O)$, $H_{28}(O)$ = observed 7- and 28-day heats of hydration, cal per g

$H_7(C)$, $H_{28}(C)$ = calculated 7- and 28-day heats of hydration, cal per g

y = heat of hydration, cal per g, in correlation equations

x_1 = percent C_3S or corrected percent C_3S in cement

x_2 = percent C_2S or corrected percent C_2S in cement

x_3 = percent C_3A or percent glass content in cement

x_4 = percent C_4AF in cement

$\bar{y}$, $\bar{x}_1$, $\bar{x}_2$ = mean values of y, x_1, x_2, . . .

b_1, b_2 = partial regression coefficients

c_{ij} = Gaussian multipliers

N = number of sets of data

k = number of independent variables in the correlation equation

P = probability level

t = value of t-distribution at probability level P and at degrees of freedom ϕ

F = value of F-distribution at probability level P and at degrees of freedom for greater mean square or numerator ϕ_n, and at those for the denominator ϕ_d

$s^2(y)$ = residual variance about the correlation

$s^2(y_i)$ = variance of the predicted value, y_i

From a preliminary analysis, with the specific surface of the cements and their C_3S, C_2S, C_3A, and C_4AF content (calculated by the Bogue method) as the independent variables, it has been found that the factor of specific surface need not be considered. The specific surface of the cement is not significantly influential to the development of heat of hydration at 7 and 28 days in the range studied.

The general linear multiple correlation equation that is to be fitted with data is

$$y = \bar{y} + b_1(x_1 - \bar{x}_1) + b_2(X_2 - \bar{x}_2) + b_3(x_3 - \bar{x}_3) \quad \text{.....(1)}$$

TABLE 1—HEAT OF HYDRATION AND PERCENT C_3S, C_2S, C_3A, AND C_4AF

Cement No.	$H_7(O)$ y, cal per g	$H_{28}(O)$ y, cal per g	C_3S x_1, percent	C_2S x_2, percent	C_3A x_3, percent	C_4AF x_4, percent
1	68.8	84.2	48.61	25.43	9.41	11.00
2	62.8	81.9	42.73	31.21	10.21	10.43
3	59.4	77.1	38.48	35.04	9.71	10.61
4	66.5	—	46.76	29.46	8.68	10.28
5	70.7	83.5	47.01	26.49	9.76	10.43
6	72.7	—	52.44	20.76	8.68	11.04
7	67.4	83.5	46.81	26.84	9.88	10.15
8	67.7	84.5	47.87	26.76	9.41	10.67
9	72.3	86.4	52.42	22.15	9.15	11.00
10	73.1	85.6	53.90	22.24	7.63	10.91
11	70.3	87.3	48.61	27.03	7.84	11.07
12	60.2	82.7	45.02	30.72	8.25	10.85
13	64.3	81.1	47.67	29.89	7.32	10.58
14	69.4	86.8	47.43	30.10	7.67	10.85
15	57.7	84.1	45.16	31.73	7.35	10.70
16	63.3	87.8	52.35	24.82	7.02	10.49
17	66.7	84.3	51.46	26.18	6.76	10.76
18	67.5	85.2	48.51	29.55	5.90	11.46
19	66.6	85.7	49.00	29.35	6.91	10.31
20	62.6	80.9	49.91	28.64	6.22	10.97
Mean for $H_7(O)$	66.5000	—	48.1075	27.7195	8.1880	10.7280
Mean for $H_{28}(O)$	—	84.0333	47.9417	28.0094	8.1333	10.7356

TABLE 2—HEAT OF HYDRATION AND CORRECTED PERCENT C_3S, C_2S, AND GLASS

Cement No.	$H_7(O)$, y, cal per g	$H_{28}(O)$, y, cal per g	Corrected C_3S x_1, percent	Corrected C_2S x_2, percent	Glass content x_3, percent
1	68.8	84.2	49.89	20.05	24.51
2	62.8	81.9	44.78	25.12	24.68
3	59.4	77.1	40.12	29.37	24.35
4	66.5	—	47.88	24.52	22.78
5	70.7	83.5	48.76	20.75	24.18
6	72.7	—	53.22	15.95	23.74
7	67.4	83.5	48.77	20.96	23.96
8	67.7	84.5	49.29	21.33	24.09
9	72.3	86.4	53.53	16.97	24.22
10	73.1	85.6	54.06	18.22	22.40
11	70.3	87.3	48.84	22.88	22.84
12	60.2	82.7	45.61	26.21	23.02
13	64.3	81.1	47.77	26.05	21.64
14	69.4	86.8	47.64	26.04	22.37
15	57.7	84.1	45.23	27.89	21.82
16	63.3	87.8	52.30	21.20	21.18
17	66.7	84.3	51.10	22.79	21.26
18	67.5	85.2	47.35	26.90	21.17
19	66.6	85.7	48.96	25.78	20.83
20	62.6	80.9	49.14	25.68	20.91
Mean for $H_7(O)$	66.5000	—	48.7120	23.2330	22.7975
Mean for $H_{28}(O)$	—	84.0333	48.5078	23.5661	22.7461

Method I

For the multiple correlations between H_7 and H_{28} and C_3S, C_2S, C_3A, and C_4AF, the following sets of simultaneous equations derived from data in Table 1, are to be solved to determine the partial regression coefficients, b_1, b_2, ..., using Gaussian multipliers:[7,8]

For H_7

$$\begin{aligned} 250.1092\,b_1 - 213.5411\,b_2 - 39.2552\,b_3 + 7.4638\,b_4 &= 200.3030 \\ -213.5411\,b_1 + 240.3033\,b_2 - 2.6100\,b_3 - 5.5505\,b_4 &= -236.1930 \\ -39.2552\,b_1 - 2.6100\,b_2 + 32.4401\,b_3 - 3.2812\,b_4 &= 13.8420 \\ 7.4638\,b_1 - 5.5505\,b_2 - 3.2812\,b_3 + 2.0043\,b_4 &= 7.4090 \end{aligned}$$

For H_{28}

$$\begin{aligned} 228.7387\,b_1 - 180.1783\,b_2 - 40.8870\,b_3 + 5.5309\,b_4 &= 116.9080 \\ -180.1783\,b_1 + 187.3261\,b_2 + 0.2430\,b_3 - 2.6388\,b_4 &= -94.8017 \\ -40.8870\,b_1 + 0.2430\,b_2 + 31.9022\,b_3 - 3.2068\,b_4 &= -18.0060 \\ 5.5309\,b_1 - 2.6388\,b_2 - 3.2068\,b_3 + 1.7052\,b_4 &= 3.1037 \end{aligned}$$

The Gaussian multipliers for which are:

For H_7

c	i1	i2	i3	i4
1*j*	0.1995	0.1856	0.2794	0.2285
2*j*	0.1854	0.1771	0.2617	0.2282
3*j*	0.2793	0.2618	0.4290	0.3873
4*j*	0.2285	0.2284	0.3873	0.9146

For H_{28}

c	i1	i2	i3	i4
1*j*	0.3638	0.3534	0.4932	0.2944
2*j*	0.3534	0.3488	0.4801	0.2963
3*j*	0.4932	0.4801	0.7074	0.4736
4*j*	0.2943	0.2963	0.4736	0.9809

From these Gaussian multipliers and the values of $\Sigma' x_i y$, the following partial regression coefficients are obtained:

For H_7

$$b_1 = 1.6870, \quad b_2 = 0.6378, \quad b_3 = 2.9181, \quad b_4 = 3.9667$$

For H_{28}

$$b_1 = 1.0598, \quad b_2 = 0.5239, \quad b_3 = 0.8739, \quad b_4 = 0.8370$$

Substituting the appropriate values of $\bar{y}$, $\bar{x}_1$, $\bar{x}_2$, ..., and the partial regression coefficients into Eq. (1), the following linear multiple correla-

TABLE 3 — VARIANCE ANALYSIS OF H_7, METHOD I

Source	Sum of squares	Degrees of freedom	Mean square	F
Due to regression	257.0489	4	64.2622	8.09
About regression	119.1911	15	7.9461	
Total	376.2400	19		

TABLE 4 — VARIANCE ANALYSIS OF H_{28}, METHOD I

Source	Sum of squares	Degrees of freedom	Mean square	F
Due to regression	61.0949	4	15.2737	3.46
About regression	57.3251	13	4.4096	
Total	118.4200	17		

TABLE 5 — VARIANCE ANALYSIS OF H_7, METHOD II

Source	Sum of squares	Degrees of freedom	Mean square	F
Due to regression	249.8273	3	83.2758	10.54
About regression	126.4127	14	7.9008	
Total	376.2400	17		

TABLE 6 — VARIANCE ANALYSIS OF H_{28}, METHOD II

Source	Sum of squares	Degrees of freedom	Mean square	F
Due to regression	60.4042	3	20.1347	4.86
About regression	58.0157	14	4.4140	
Total	118.4200	17		

TABLE 7 — DATA FOR CALCULATION OF PREDICTED VALUES

Cement No.	C_3S	C_2S	C_3A	C_4AF	Corrected C_3S	Corrected C_2S	Glass content
1	46.39	32.09	5.19	11.55	44.77	29.98	20.47
2	41.89	35.46	7.01	11.13	41.53	31.94	22.02
3	38.17	39.51	7.11	11.95	35.54	36.04	23.16
4	40.04	37.71	6.61	11.86	39.14	34.60	22.48
5	29.71	47.51	6.96	11.64	29.11	44.11	22.60
6	45.27	31.56	7.10	11.49	44.82	28.03	22.57
7	51.76	27.18	5.80	11.16	50.66	24.56	20.67
8	50.33	28.49	5.86	11.34	49.20	25.85	20.97
9	52.35	26.25	4.98	11.07	49.46	24.68	20.51
10	55.65	23.13	5.23	11.04	54.26	20.92	19.87

tion equations between 7- and 28-day heats of hydration and compound composition are obtained:

For H_7

$$y = 1.6870x_1 + 0.6378x_2 + 2.9181x_3 + 3.9667x_4 - 98.7851 \quad \text{(2)}$$

For H_{28}

$$y = 1.0598x_1 + 0.5239x_2 + 0.8739x_3 + 0.8370x_4 + 2.4572 \quad \text{(3)}$$

The significance of the correlation can be tested by an analysis of variance shown in Tables 3 and 4.

Since the calculated value of $F(8.09)$ is greater than 4.89, the tabulated value of F-distribution at $P = 0.01$ and $\phi_n = 4$ and $\phi_d = 15$, the correlation is significant at the indicated probability level of 1 percent.

Since the calculated value of $F(3.46)$ is greater than 3.18, the tabulated value of F-distribution at $P = 0.05$ and $\phi_n = 4$ and $\phi_d = 13$, the correlation is significant at the indicated probability level of 5 percent.

Method II

For the multiple correlations between H_7 and H_{28} and corrected C_3S and C_2S and glass content, the following sets of simultaneous equations are to be solved to determine the partial regression coefficients, b_1, b_2, ..., using Gaussian multipliers:

For H_7

$$\begin{aligned} 208.3147\,b_1 - 198.9625\,b_2 - 13.1385\,b_3 &= 206.0230 \\ -198.9625\,b_1 + 262.1028\,b_2 - 34.7029\,b_3 &= -245.5390 \\ -13.1385\,b_1 - 34.7029\,b_2 + 34.4822\,b_3 &= 24.8990 \end{aligned}$$

For H_{28}

$$\begin{aligned} 186.5497\,b_1 - 163.8355\,b_2 - 17.5908\,b_3 &= 103.9044 \\ -163.8355\,b_1 + 205.4070\,b_2 - 27.5080\,b_3 &= -80.3627 \\ -17.5908\,b_1 - 27.5080\,b_2 + 33.5460\,b_3 &= -16.2817 \end{aligned}$$

The Gaussian multipliers are:

For H_7

c	i1	i2	i3
1*j*	0.1950	0.1822	0.2576
2*j*	0.1822	0.1745	0.2451
3*j*	0.2577	0.2451	0.3738

For H_{28}

c	i1	i2	i3
1*j*	0.2825	0.2754	0.3740
2*j*	0.2754	0.2740	0.3691
3*j*	0.3739	0.3690	0.5285

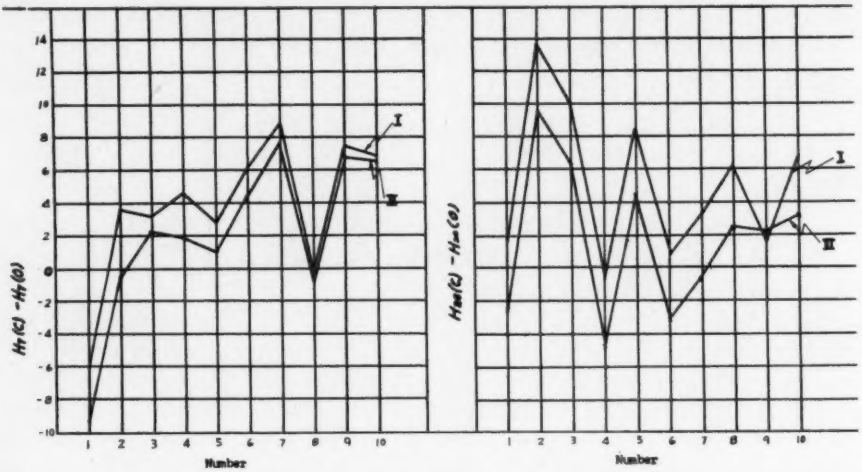

Fig. 1—Difference of calculated and observed heats of hydration of portland cement at 7 and 28 days

From these Gaussian multipliers and the values of $\Sigma' x_1 y$, the following partial regression coefficients may be obtained:

For H_7

$$b_1 = 1.8682, \quad b_2 = 0.7746, \quad b_3 = 2.2142$$

For H_{28}

$$b_1 = 1.1313, \quad b_2 = 0.5911, \quad b_3 = 0.5922$$

Substituting the appropriate values $\bar{y}$, $\bar{x}_1$, $\bar{x}_2, \ldots$, and the partial regression coefficients into Eq. (1), the following linear multiple correlations between 7- and 28-day heats of hydration and corrected C_3S, C_2S, and glass content are obtained:

For H_7

$$y = 1.8682x_1 + 0.7746x_2 + 2.2142x_3 - 92.9783 \quad \ldots\ldots (4)$$

For H_{28}

$$y = 1.1313x_1 + 0.5911x_2 + 0.5922x_3 - 1.7554 \quad \ldots\ldots (5)$$

The significance of the correlations can be tested by an analysis of variance shown in Tables 5 and 6.

Since the calculated value of $F(10.54)$ is greater than 5.29, the tabulated value of F-distribution at $P = 0.01$, and $\phi_n = 3$ and $\phi_d = 14$, the correlation is significant at the indicated probability level of 1 percent.

Since the calculated value of $F(4.86)$ is greater than 4.37, the tabulated value of F-distribution at $P = 0.025$, and $\phi_n = 3$ and $\phi_d = 14$, the correlation is significant at the indicated probability level of 2.5 percent.

DISCUSSION

Corresponding to any set of given values of the independent variables, i.e., C_3S, C_2S, C_3A, and C_4AF; or corrected C_3S, corrected C_2S, and glass content; there is a predicted value of heat of hydration, that can be calculated from the correlation Eq. (2)-(5). The predicted value of heat of hydration thus calculated is subject to uncertainty since it is derived by using coefficients that are subject to uncertainty. This uncertainty represents the confidence range of the predicted value of heat of hydration of a cement. It can be determined by first calculating the variance of y_i, the predicted value of heat of hydration,[8] thus

TABLE 8—PREDICTED VALUES COMPUTED FROM CORRELATION EQ. (2)-(5)

Cement No.	$H_7(C) \pm ts(y_i)$ Method I, p=0.01	$H_7(C) \pm ts(y_i)$ Method II, P=0.01
1	48.12 ± 13.9007	45.61 ± 11.9151
2	55.58 ± 12.1676	52.61 ± 8.0391
3	59.10 ± 5.7755	58.11 ± 4.8403
4	59.15 ± 10.1730	56.72 ± 6.7423
5	60.90 ± 7.4930	59.21 ± 5.3029
6	64.01 ± 5.9040	62.44 ± 3.3104
7	66.37 ± 4.8950	65.39 ± 3.7552
8	67.06 ± 4.2732	66.46 ± 4.0315
9	64.72 ± 6.8362	63.95 ± 5.0954
10	68.90 ± 5.6833	68.59 ± 5.7042
Cement No.	$H_{28}(C) \pm ts(y_i)$ Method I, P=0.05	$H_{28}(C) \pm ts(y_i)$ Method II, P=0.025
1	74.66 ± 7.8259	70.63 ± 8.0490
2	77.71 ± 6.9766	73.47 ± 5.1422
3	80.87 ± 3.2188	77.15 ± 3.3479
4	80.35 ± 5.7221	76.29 ± 5.1979
5	82.64 ± 4.4680	78.74 ± 3.2522
6	82.79 ± 3.2359	78.88 ± 2.3845
7	85.34 ± 2.6937	81.60 ± 2.5588
8	85.96 ± 2.3425	82.32 ± 2.4593
9	80.39 ± 4.5412	80.93 ± 3.4631
10	87.36 ± 4.4554	83.76 ± 3.8531

$$s^2(y_i) = s^2(y)\left[\frac{1}{N} + c_{11}(x_1 - \bar{x}_1)^2 + c_{22}(x_2 - \bar{x}_2)^2 + \cdots + 2c_{12}(x_1 - \bar{x}_1)(x_2 - \bar{x}_2) + 2c_{13}(x_1 - \bar{x}_1)(x_3 - \bar{x}_3) + \cdots\right]$$

The standard error of y_i is $\pm\sqrt{s^2(y_i)}$, and the confidence range at the probability level of significance of correlation is found by multiplying it with the appropriate value of the t-distribution at $N - k - 1$ degrees of freedom. Then the predicted value and its confidence range are given by $y_i \pm ts(y_i)$.

With further data, Table 7, the predicted values are calculated from the correlation Eq. (2)-(5), and the confidence range for each is calculated as above, at the probability level of significance of correlation. The results of such calculations are as shown in Table 8.

From Table 8 it can be seen that, in the long run, the confidence ranges as calculated by Method II are narrower than those by Method I, that is, the precision of estimation is greater by using Method II. In Table 9 and Fig. 1, the estimated values are compared with the corresponding observed values. The differences of $H_7(C)\,\mathrm{II} - H_7(O)$ and $H_{28}(C)\,\mathrm{II} - H_{28}(O)$ compare favorably with those of $H_7(C)\,\mathrm{I} - H_7(O)$ and $H_{28}(C)\,\mathrm{I} - H_{28}(O)$, respectively, although analyses of variance show that the respective correlations are significant at about the same

TABLE 9 — DIFFERENCE OF CALCULATED AND OBSERVED HEATS OF HYDRATION OF PORTLAND CEMENT AT 7 AND 28 DAYS

Cement No.	$H_7(C)I$	$H_7(C)II$	$H_7(O)$	$H_7(C)I - H_7(O)$	$H_7(C)II - H_7(O)$
1	48.12	45.61	54.8	−6.68	−9.19
2	55.58	52.61	52.0	3.58	0.61
3	59.10	58.11	55.9	3.20	2.21
4	59.15	56.72	54.8	4.35	1.92
5	60.90	59.21	58.2	2.70	1.01
6	64.01	62.44	58.0	6.01	4.44
7	66.37	65.39	57.7	8.67	7.69
8	67.06	66.46	67.3	−0.24	−0.84
9	64.72	63.95	57.2	7.52	6.75
10	68.90	68.59	62.0	6.90	6.59
Cement No.	$H_{28}(C)I$	$H_{28}(C)II$	$H_{28}(O)$	$H_{28}(C)I - H_{28}(O)$	$H_{28}(C)II - H_{28}(O)$
1	74.66	70.63	73.3	1.36	−2.67
2	77.71	73.47	64.0	13.71	9.47
3	80.87	77.15	70.9	9.97	6.25
4	80.35	76.29	81.1	−0.75	−4.81
5	82.64	78.74	74.3	8.34	4.44
6	82.79	78.88	82.1	0.69	−3.22
7	85.34	81.60	82.2	3.14	−0.60
8	85.96	82.32	79.9	6.06	2.42
9	80.39	80.93	78.7	1.69	2.23
10	87.36	83.76	80.7	6.66	3.06

level of probability. In other words, 7- and 28-day heats of hydration can be more reliably predicted using the correlation equations derived by Method II.

CONCLUSIONS

An attempt is made to more reliably and precisely predict the 7- and 28-day heats of hydration of the portland cement as produced at this plant. Linear multiple correlation equations are derived by taking: (1) the C_3S, C_2S, C_3A, and C_4AF as the independent variables (Method I), (2) the corrected C_3S and C_2S and glass content as the independent variables (Method II). Estimations calculated from these equations are compared with the observed heats of hydration. It has been found that, although analyses of variance show that these respective correlations are both significant at the same probability levels, the 7- and 28-day heats of hydration can be more reliably estimated using the correlation equations with the corrected C_3S and C_2S and glass content as the independent variables.

The precision of estimation as shown by the confidence ranges of predicted values, is, in the long run, also better when calculated by Method II.

Under normal conditions of production at this plant, specific surface of the portland cement has been found not to be significantly influential on the 7- and 28-day heats of hydration.

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Calculo de las Temperaturas de Hidratacion del Cemento Portland

Se estudian por análisis estadístico dos métodos para estimar las temperaturas de hidratación del cemento portland por 7 y 28 dias, tomando por base datos analíticos conocidos. En el método I los calores de hidratación están correlacionados con el contenido calculado del compuesto, es decir, C_3S, C_2S, C_3A, y C_4AF como las variables independientes y en el método II, están correlacionados con los C_3S y C_2S corregidos y el contenido de vidrio, calculado de acuerdo con las series de ecuaciones derivadas de Dahl, como las variables independientes. Las comparaciones de los valores observados demuestran que los valores predichos calculados por el método II se comparan favorablemente con aquellos obtenidos por el método I de modo que los cálculos estimados se puedan hacer más exactos y seguros.

Estimation de la Chaleur d'Hydratation du Cément Portland

Deux méthodes d'estimation de 7 à 28 jours de chaleurs d'hydratation du cément portland de la part des données analytiques, sont étudiées par l'analyse statistique. Dans la Méthode I, les chaleurs d'hydratation sont corrélatives avec le contenu composé calculé, ça veut dire, C_3S, C_2S, C_3A, et C_4AF, comme le variables indépendants; et dans la Méthode II, elles sont corrélatives avec les C_3S et C_2S corrigés et le contenu de verre, d'après le calcul par les séries d'équations provenues par Dahl comme les variables indépendants. Les comparaisons avec les valeurs observées montrent que les valeurs prédites, calculées par la Méthode II, se confrontent favorablement avec celles par la Méthode I, de sorte qu'on peut faire estimations plus exactes et dignes de confiance.

Bestimmung der Hydrationswärme von Portland-Zement

Zwei Bestimmungsmethoden für 7- und 28-tägige Hydrationswärmen von Portland-Zement aus analytischen Angaben werden mittels statistischer Analyse studiert. In Methode I werden die Hydrationswärmen mit dem errectneten Gemischgehalt, d.h. C_3S, C_2S, C_3A und C_4AF als die unabhängigen Veränderlichen in Beziehung gebracht, und in Methode II werden die Hydrationswärmen mit dem berichtigten C_3S, C_2S und Glasgehalt, wie mittels der von Dahl abgeleitteten Serie von Gleichungen errechnet, als die unabhängigen Veranderlichen in Beziehung gebracht. Vergleiche mit festgestellten Werten zeigen, dass vorausgesagte, durch Methode II errechnete Werte sich gut mit denen der Methode I vergleichen lassen, so dass Bestimmungen genauer und verlässlicher gemacht werden können.

CONCRETE BRIEFS

Notes from Field and Office

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Characteristic Equation of Cylindrical Shells--A Simplified Method of Solution

By G. S. RAMASWAMY* and M. RAMAIAH†

THE EIGHTH DEGREE algebraic equation associated with the eighth order partial differential equation occurring in the solution of cylindrical shells is solved by a new and simplified method. The method is based on Steinman's rule for determining the real roots of higher degree equations.

The analysis of cylindrical shells by any of the well-known theories involves the solution of a homogeneous partial differential equation of the eighth order. The auxiliary algebraic equation associated with it contains only the even powers and has eight complex roots occurring in four conjugate pairs, the roots being symmetrical about the real and imaginary axes. The roots can be written down explicitly only in the case of the Jenkins' and the Schorer's forms of the characteristic equation. In all other cases, the existing methods of determining the roots are tedious and involve several operations with complex numbers. The method presented here considerably simplifies the calculations involved without any sacrifice of accuracy. It is an extension of Steinman's method[1] for determining the real roots of higher degree equations.

This method has already been presented by one of the authors elsewhere[2] in its preliminary form. At that stage, its general applicability over the entire range of short, intermediate, and long shells was not established, as not enough examples covering a wide range of shell dimensions had been worked out. Moreover, certain generalized conclusions, which are presented here, could not then be drawn.

REVIEW OF EXISTING METHODS

Two methods are in current use for determining the roots of the characteristic equation.

Method I

In this method, the eighth degree equation is first transformed into a biquadratic; the biquadratic is in turn successively recast as a coupled quadratic

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and an ordinary quadratic equation, which is then solved. Working backwards, the roots of the eighth degree equation are obtained. This method, described in detail by Lundgren,[3] is essentially an iterative procedure and at least two cycles are necessary to get reasonably accurate results. The method is tedious as it involves operations with complex numbers at all stages.

Method 2

The other method is to convert the eighth degree equation into a biquadratic and then form its cubic resolvent equation, which is then usually solved by trigonometric methods.[4] Here again, the method tends to be tedious and lengthy.

SUMMARY OF THE NEW METHOD

The new method is identical with Method 2 to the point where the resolvent cubic equation is formed. The roots of the cubic equation are, however, extracted directly by Steinman's rule, which is given below:

Steinman's rule: If the equation is

$$mx^n = ax^{n-1} + bx^{n-2} + cx^{n-3} + dx^{n-4} + \ldots$$

and, if x_o is an approximate value of one of its real roots, then its exact value x_1 is given by

$$x_1 = \frac{a + \dfrac{2b}{x_o} + \dfrac{3c}{x_o^2} + \dfrac{4d}{x_o^3} + \ldots}{m + \dfrac{b}{x_o^2} + \dfrac{2c}{x_o^3} + \dfrac{3d}{x_o^4} + \ldots}$$

The following generalized conclusions, based on the analysis of a large number of shells over the entire range of long, intermediate and short shells, further simplify the calculations:

(a) The resolvent cubic equation, to which the eighth degree equation is progressively reduced, has three real roots.

(b) One of the roots is nearly zero and can be ignored without any loss of accuracy.

(c) The other two roots are nearly equal to +2 and −2, respectively.

As these two values can be taken as the trial roots for insertion in Steinman's formula, a high degree of accuracy is possible with only two trials. At this stage, it is easy to verify that the third root is practically zero. The method is best understood by following through a worked example. The example presented here refers to a long shell; but it is found by the authors, by working out a large number of examples, that the method is applicable to intermediate and short shells. The Lundgren characteristic equation is used here for purposes of illustration. The method, however, is equally applicable to the solution of the characteristic equations associated with other theories.

EXAMPLE

The following dimensions were used to solve the characteristic equation of the shell: Length = 80 ft, radius = 24 ft, thickness = 3 in. The shell parameters ρ and k are calculated as follows:

$$\rho = \sqrt[8]{\frac{12\,r^4 R^4}{l^4 t^8}} = 4.1466$$

$$k = \frac{\pi^2 R^2}{l^2 \rho^2} = 0.0517$$

The Lundgren characteristic equation is given below:

$$m^8 + \left[\frac{2}{\rho^2} - 4k\right] m^6 + \left[\frac{1}{\rho^4} - (6 + \nu)\frac{k}{\rho^2} + 6k^2\right] m^4 + \left[-(2 + \nu)\frac{k}{\rho^4} + (3 + 2\nu + \nu^2)\frac{k^2}{\rho^2} - 4k^3\right] m^2 + \left[\left(-\nu\frac{k^2}{\rho^2} + k^4\right) + (1 - \nu^2)\right] = 0$$

where ν is Poisson's ratio and is assumed as 0.2. The characteristic equation for the given shell is formed and is given below:

$$m^8 - 0.090492\, m^6 + 0.000778\, m^4 - 0.0004028\, m^2 + 0.9600055 = 0 \quad \ldots\ldots(1)$$

Putting $m^2 = y$, this is reduced to

$$y^4 - 0.090492\, y^3 + 0.000778\, y^2 - 0.0004028\, y + 0.9600055 = 0 \quad \ldots\ldots(2)$$

This is of the form $y^4 + py^3 + qy^2 + ry + s = 0$, where $p = -0.090492$, $q = +0.000778$, $r = -0.0004028$, and $s = +0.9600055$.

By substituting $y = (x - p/4)$, this is reduced to the form $x^4 + ax^2 + bx + c = 0$. Making this substitution, the equation reduces to

$$x^4 - 0.002293\, x^2 - 0.00046023\, x + 0.959996 = 0 \quad \ldots\ldots(3)$$

The resolvent cubic equation for Eq. (3) is of the form $Z^3 + 2aZ^2 + (a^2 - 4c)Z - b^2 = 0$, or

$$Z^3 - 0.004586\, Z^2 - 3.83998\, Z - 0.0000002118 = 0 \quad \ldots\ldots(4)$$

$$\therefore Z^3 = +0.004586\, Z^2 + 3.83998\, Z + 0.0000002118$$

It is easily seen that the approximate values of two of the three roots are $+2$ and -2 and that the third root is small. By using $+2$ and -2 as the trial roots, their exact values are found from Steinman's formula.

First root: Assume $Z_0 = +2$ then

$$Z_1 = \frac{+0.004586 + \frac{2 \times 3.83998}{2} + \frac{3 \times 0.0000002118}{2^2}}{1 + \frac{3.83998}{2^2} + \frac{2 \times 0.0000002118}{2^3}}$$

$$= +1.961518297$$

A second trial is made using $Z_0 = +1.961518297$.

$$Z_1 = \frac{+0.004586 + \frac{2 \times 3.83998}{1.961518297} + \frac{3 \times 0.0000002118}{(1.961518297)^2}}{1 + \frac{3.83998}{(1.961518297)^2} + \frac{2 \times 0.0000002118}{(1.961518297)^3}}$$

$$= +1.961881$$

Second root: Assume $Z_0 = -2$, then

$$Z_2 = \frac{+0.004586 + \frac{2 \times 3.83998}{(-2)} + \frac{3 \times 0.0000002118}{(-2)^2}}{1 + \frac{3.83998}{(-2)^2} + \frac{2 \times 0.0000002118}{(-2)^3}}$$

$$= -1.956838765$$

Another trial is made using $Z_0 = -1.956838765$ and Z_2 is found out to be equal to -1.957295. $(Z - Z_1)(Z - Z_2) = (Z - 1.961881)(Z + 1.957295) = (Z^2 - 0.004586Z - 3.83997987)$. Comparing this with Eq. (4), it is easily seen that the third root can be taken as zero.

The three roots of the resolvent cubic are, therefore, $Z_1 = +1.961881$, $Z_2 = -1.957295$, and $Z_3 = 0$.

The roots of Eq. (3) are given by Descartes as shown below:

$$x_1 = \tfrac{1}{2}\left[+\sqrt{Z_1} + (\sqrt{Z_2} + \sqrt{Z_3})\right]$$

$$x_2 = \tfrac{1}{2}\left[+\sqrt{Z_1} - (\sqrt{Z_2} + \sqrt{Z_3})\right]$$

$$x_3 = \tfrac{1}{2}\left[-\sqrt{Z_1} + (\sqrt{Z_2} - \sqrt{Z_3})\right]$$

$$x_4 = \tfrac{1}{2}\left[-\sqrt{Z_1} - (\sqrt{Z_2} - \sqrt{Z_3})\right]$$

Therefore,

$$x_1 = \tfrac{1}{2}[+1.4006716 + i\,1.3990337]$$

$$x_2 = \tfrac{1}{2}[+1.4006716 - i\,1.3990337]$$

$$x_3 = \tfrac{1}{2}[-1.4006716 + i\,1.3990337]$$

$$x_4 = \tfrac{1}{2}[-1.4006716 - i\,1.3990337]$$

We know $y = (x - p/4) = x + 0.090492/4$. Therefore, adding the value of $(-p/4)$ to the real parts of the above roots, the roots of Eq. (2) are given by

$$y_1 = \tfrac{1}{2}[+1.4459176 + i\,1.3990337]$$

$$y_2 = \tfrac{1}{2}[+1.4459716 - i\,1.3990337]$$

$$y_3 = \tfrac{1}{2}[-1.3554256 + i\,1.3990337]$$

$$y_4 = \tfrac{1}{2}[-1.3554256 - i\,1.3990337]$$

$$m^2 = y \quad \text{or,} \quad m = \pm\sqrt{y}$$

The square roots of complex numbers are now given:

$$\sqrt{g + ih} = \pm\sqrt{\tfrac{1}{2}}\left[\sqrt{\sqrt{g^2 + h^2} + g} + i\sqrt{\sqrt{g^2 + h^2} - g}\right]$$

and,

$$\sqrt{g - ih} = \pm\sqrt{\tfrac{1}{2}}\left[\sqrt{\sqrt{g^2 + h^2} + g} - i\sqrt{\sqrt{g^2 + h^2} - g}\right]$$

Using the above formulas, the eight roots of Eq. (1) are given by

$$m_{1,2,3,4} = \pm\tfrac{1}{2}\left[\sqrt{\sqrt{(1.4459176)^2 + (1.3990337)^2} + 1.4459176} \pm i\sqrt{\sqrt{(1.4459176)^2 + (1.3990337)^2} - 1.4459176}\right]$$

and,

$$m_{4,5,7,8} = \pm\tfrac{1}{2}\left[\sqrt{\sqrt{(-1.3554256)^2 + 1.3990337)^2} - 1.3554256} \pm i\sqrt{\sqrt{(-1.3554256)^2 + (1.3990337)^2} + 1.3554256}\right]$$

Simplifying, the eight roots are now found: $m_{1,2,3,4} = \pm(0.929769 \pm i\,0.376178)$, and $m_{4,5,7,8} = \pm\,(0.3848747 \pm i\ 0.908758)$.

The product of the eight roots equals the constant term of Eq. (1). The product is $(0.929769^2 + 0.376178^2)^2(0.3848747^2 + 0.908758^2)^2$. This works out to 0.9599968, or 0.96 as against 0.9600055.

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Design Aids for the Ultimate Moment Capacity of Beams and Slabs

By FRITZ KRAMRISCH*

THE ULTIMATE MOMENT CAPACITY of a rectangular beam controlled by tension is governed by Eq. (A1), Section A605, Appendix to the ACI Building Code (ACI 318-56).

$$M_u = bd^2 f_c' q\,(1 - 0.59\,q)$$

In practical design work the qualities of the concrete and of the reinforcing steel are usually constant, at least for a greater part of the work. By introducing these values into Eq. (A1), and by transforming the ultimate moment capacity into ft-kips, we obtain $q = pf_y/f_c'$, where $f_y = 40$ kips per sq in. for intermediate grade steel, and $f_c' = 3$ kips per sq in. Let $b = 12$ in. for slabs and 10 in. for beams.

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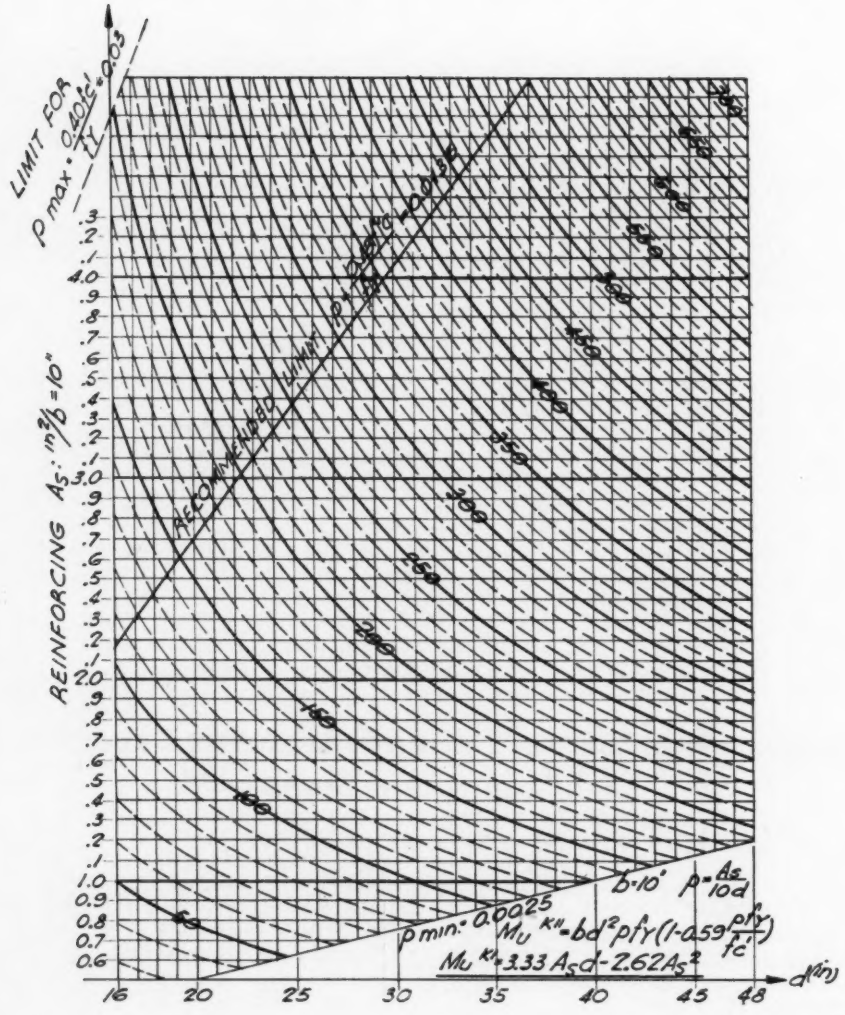

Fig. 1—Ultimate moment capacity M_u (ft-kips) for 10 in. wide beams, where $f_y = 40$ kips per sq in., and $f_c' = 3$ kips per sq in.

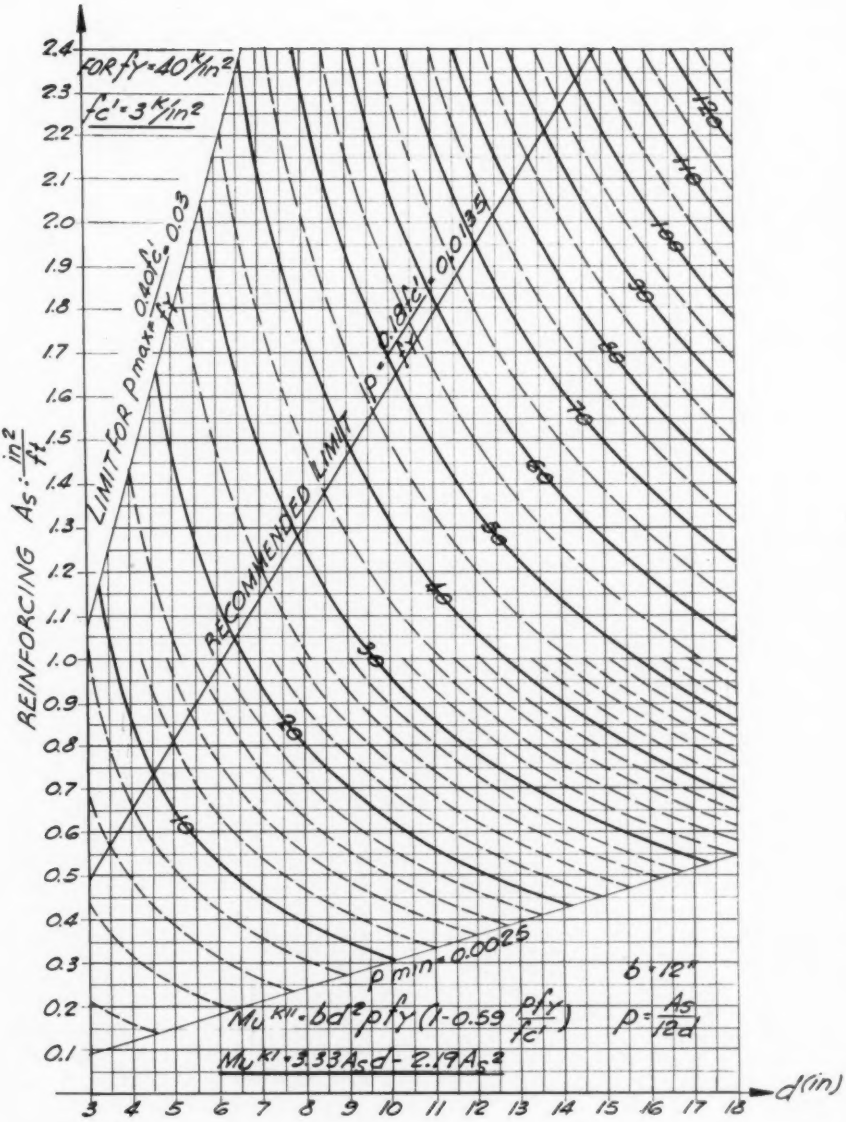

Fig. 2—Ultimate moment capacity M_u (ft-kips) for 12 in. wide slabs, where $f_y = 40$ kips per sq in., and $f_c' = 3$ kips per sq in.

Slabs:

$$M_u(\text{ft-kips}) = 3.33\,A_s d - 2.19\,A_s^2 \quad (1)$$

Beams:

$$M_u(\text{ft-kips}) = 3.33\,A_s d - 2.62\,A_s^2 \quad (2)$$

The equations are plotted in Fig. 1 and 2. These diagrams do not require the calculation of special values to enter the graph. They are directly applicable and offer also an important visual aid to the designer because they show the location of the selected section with respect to the recommended limits regarding deflection and minimum or maximum rate of reinforcing.

EXAMPLES

1. 6½-in. slab, $M_u = 7$ ft-kips, $d = 5.5$ in., $A_s = 0.7$ sq in. per ft.

2. 14 × 26-in. beam, $M_u = 250$ ft-kips, $d = 23.5$ in.; for a 10 in. wide section, $M_u = 250 \times 10/14 = 179$ ft-kips, $A_s = 2.5$ sq in. Therefore, total reinforcement $A_s = 2.5 \times 14/10 = 3.5$ sq in.

A Look at Column Load Capacities

By H. C. PFANNKUCHE*

FOR SOME TIME it has been apparent that a re-examination of the capacities of conventionally designed spirally reinforced concrete columns is in order. Especially has this been the case since ASTM approved the Tentative Specifications A 431 and A 432 covering high strength billet bars for concrete reinforcement with minimum yield strengths of 75,000 and 60,000 psi, respectively.

While this paper does not pretend to cover all the possibilities available for increasing load carrying capacities of spirally reinforced concrete columns, its purpose is to stimulate interest in methods which will permit the designer to maintain minimum column sizes.

The existing ACI Building Code (ACI 318-56) permits a working steel stress of 40 percent of the yield point which would establish a working stress of 30,000 psi for the A 431 bar and 24,000 psi for the A 432 bar.

Tabulated loads

Special reference is given to Table 20 in the ACI *Reinforced Concrete Design Handbook*† in which the notation at the bottom of the table, identified by double asterisk, stipulates that the tabulated maximum loads are based on one ring of maximum size bars in one outer ring. Too often engineers accept the tabulated values as the maximum without exploring other possibilities for maintaining concrete cross section and at the same time developing substantial increases in column capacity.

To illustrate this point, this paper will deal with a 24 x 24-in. column, using 3000-psi concrete, and will point out the wide range of calculated column capacities beyond the value shown in Table 20 as maximum for that par-

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†ACI Committee 317; *Reinforced Concrete Design Handbook*, American Concrete Institute, Detroit, 1955.

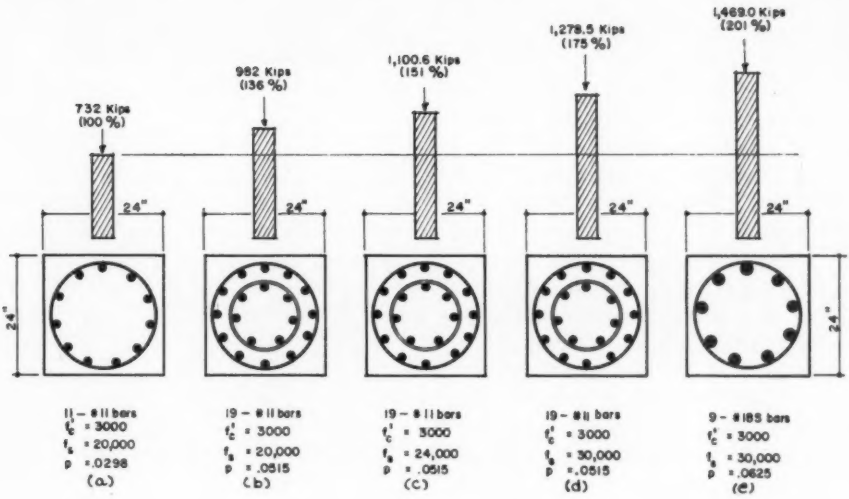

Fig. 1—Load carrying capacities of spirally reinforced concrete columns

ticular column. This situation is not peculiar to a column of this particular size; the situation is the same, more or less, with columns of other dimensions.

Consider first the loads tabulated in Table 20, using a specified concrete strength f_c' of 3000 psi and a steel working stress, f_s of 20,000 psi. The tabulated load on the concrete is 389.0 kips and the tabulated load on the bars is 343.0 kips for a total of 732.0 kips (see Fig. 1a). Reference to the upper portion of Table 21 (in the Handbook) shows that a load of 343.0 kips would be carried by 11 #11 bars. The steel percentage in this case would be about 2.98 percent.

The center portion of Table 21 would indicate that a 24 x 24-in. column with a 21 in. diameter core would accommodate 12 #11 bars in an outer ring and 7 #11 bars in the inner ring for a total of 19, for a steel ratio of about 5.15 percent. In this case, the allowable load on the concrete would remain the same at 389.0 kips but the allowable load on the bars would be increased to 593.0 kips for a total capacity of 982.0 kips on the column (see Fig. 1b). This would be an increase over the loads shown in Table 20 of 250.0 kips or about 36 percent.

Further increase

A further increase could be accomplished by maintaining the same concrete section and the same steel ratio but by using A 432 steel and increasing the working stress to 24,000 psi. This would increase the value to be assigned to the bars to 711.6 kips, bringing the total capacity of the column to 1100.6 kips (see Fig. 1c), an increase over the value shown in Table 20 of 368.6 kips or 51 percent.

By specifying that A 431 steel be used and taking advantage of a working stress of 30,000 psi, the load on the bars can be increased to 889.5 kips and the carrying capacity of the column can thus be increased to 1278.5 kips (see Fig. 1d), an increase over the Table 20 tabulated load of 546.5 kips or about 75 percent.

The Problems and Practices section (which appears in Concrete Briefs section as space allows) is an open forum on everyday topics. JOURNAL readers are invited to contribute their solutions to problems that occur in the field or office—whether common or unusual.

Items appearing in this section can vary from 100 to 1000 words. It is an ideal place for those practical items generally considered too short to be submitted for publication, but containing material of potential importance to a great many.

Perhaps YOUR solution is a better way of handling some everyday problem.

Now a steel ratio of 5.15 percent is not excessive when it is considered that the ACI Building Code permits a ratio as high as 8 percent. The Code permits point contact between vertical column bars and horizontal beam bars. Section 1103(c)3 of the Code permits welded splices and suggests that welded splices are preferable in the case of bars larger than #11. This section of the Code does not require that the splice develop the full strength of the bar but rather only the full yield strength.

The cases heretofore cited are not the maximum in any sense of the word. For example, the allowable load on the bars can be increased to 1080.0 kips with a subsequent total column capacity of 1469.0 kips (see Fig. 1e) for an increase over the value shown in Table 20 of 737.0 kips or 101 percent. This can be accomplished by using nine #18S bars in one outer ring and specifying A 431 bars and using a working stress of 30,000 psi. In this case the steel ratio would be about 6.25 percent.

It is entirely possible that the designer might find it feasible and desirable to detail steel placement, especially if beams framing into such columns were to be unusually heavily reinforced. The use of templates for proper positioning of vertical column bars might well be in order to insure adequate clearances and to facilitate steel placement.

Bending properties

In the past there has been some reluctance to use high strength steel due to its limited bending properties. This need be of no concern in considering the use of A 431 and A 432 bars. That portion of the specifications referring to bend properties for A 432 bars is identical with the requirements for hard grade billet bars meeting A 15 requirements and the variance is only slight in the requirements for the A 431 bars.

Deliberately no attempt has been made to include comparative cost information since variations in unit costs throughout the country would tend to make such comparisons meaningless. Generally, however, the column cost per kip of load will decline as column capacity is increased. More substantial economies will result when the comparison is made to columns sized in accordance with the loads shown in Table 20.

It is obvious, however, that the use of the A 432 and A 431 bars results in an economy since an increase of 20 percent in allowable stress is available with the A 432 bar at no premium and an increase of 50 percent in allowable stress is permitted with the A 431 bar at a premium of perhaps 20 percent.

A further refinement to increase steel percentage and at the same time to reduce congestion in steel placement can be brought about by using bundled reinforcement. This procedure and test results are discussed in ASCE Paper 1818.*

*Hanson, N. W., and Reiffenstuhl, Hans, "Concrete Beams and Columns with Bundled Reinforcement," *Proceedings*, ASCE, Paper 1818, No. ST6, Oct. 1955, pp. 1818-1–1818-23.

CURRENT REVIEWS

of Significant Contributions in Foreign and Domestic Publications

Bridges

Bridges of the Orly airport road crossing (Ponts de la traversée routiere de l'aéroport d'Orly)

J. Vasseur and E. Becker, *Annales*, Institut du Bâtiment et des Travaux Publics (Paris), Supplement No. 156, Dec. 1960, pp. 1306-1328

General description of the 15 reinforced concrete and prestressed concrete bridges to carry airport roads and taxiways over the national highway traversing a portion of Orly airport in Paris.

Bridge No. 10 at Orly (Le pont No. 10 a Orly)

J. Gaillard and D. Beaulieu, *Annales*, Institut Technique du Bâtiment et des Travaux Publics (Paris), No. 145, Jan. 1960, pp. 105-131
Author's Summary

Bridge No. 10, built as part of the program for the extension of Orly Airport, will provide an overpass for the Southern Freeway.

This curved structure of prestressed concrete has a central span of 176 ft and two side spans of 126 ft each.

Some of the design requirements were:

(1) Narrow limits imposed by the longitudinal profiles

(2) Better clearances obtained by the use of a curved structure versus a skew one, and of slender intermediate supports

(3) Aesthetic considerations dictated by a privileged location; the construction confronted the contractor with some delicate problems, such as the complex shape of the formwork; the high density of reinforcement and cables; and the required high strength of the concretes.

The use of prestressed concrete provided the solution to the technical problems, while conforming to architectural requirements.

Notes on post-tensioned bridges constructed in Southern Rhodesia since 1954

A. F. Mason, *Journal*, South African Prestressed Concrete Development Group (Johannesburg), May 1960, pp. 1-32
Reviewed by D. G. Norman

Paper was read at a prestress symposium in Rhodesia and outlines construction methods and design specifications used for the superstructures.

In most cases contractors submitted designs based on the official specification rather than alternatives.

Tancarville bridge (Pont-route de Tancarville)

Annales, Institut Technique du Bâtiment et des Travaux Publics (Paris), Supplement No. 157, Jan. 1961, pp. 1-167

Seven chapters and eight annexes give complete details on the design and construction of the highway bridge at Tancarville, near the mouth of the Seine River, east of Le Havre, France. The longest suspension bridge in Europe it has a suspended span 3118 ft long with an approach on the left bank of eight spans totaling 1320 ft. The

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roadway is 40 ft wide, providing for four lanes of traffic.

The suspension towers are of reinforced concrete 410 ft high. The eight independent approach spans are of prestressed concrete as are the cable anchorages.

Construction Techniques

Movable form elements

AXEL ERSEN, *Bulletin International Association for Shell Structures* (Madrid), No. 1, 6 pp.

Describes use of movable forms in the construction of a reinforced concrete hangar. Quantities of materials and labor required are given. Forms were re-used three times.

Conveyor rides prestressed truss

Construction Equipment, V. 22, No. 3, Sept. 1960, pp. 89-92

Description of novel conveyor truss that incorporates low cost, speed, and safety to its operation. A 24-in. belt on the truss carries aggregates from a ground-level hopper to storage bins atop a batch plant. The 256 ft long supply line rises 71 ft.

Formwork for concrete construction (Schalungen im Betonbau)

KURT HAEBERLEN and FRITZ KRESS, Otto Maier Verlag, Ravensburg, Germany, 1959, 256 pp., 48 DM

Reviewed by MARY K. HURD

The authors are to be commended for a thorough and copiously illustrated treatment of both major and minor considerations in building formwork and its bracing and supporting members of wood (not including plywood). Only in the section on ribbed and waffle slabs is any major departure made from the emphasis on wooden forms. The work is significant for its attention to particulars in a field where details are too often neglected or poorly executed. Concentration is on know-how for form carpenters, but the book should be valuable to form planners, engineers, and inspectors as well.

Opening sections deal with German building code requirements for form construction, and with tools and materials available to the form builder. The various components of wood formwork are discussed one by one: wedges, and how to cut them and fasten in place; shores, and preferred ways of splicing and bracing them; stakes; bracing members; twisted wire ties; panels and how to build them from boards.

Suggestions are made for organizing a workplace for fabricating forms and components, storing materials and completed form panels or parts, cleaning form lumber after use, etc. Form planning on an over-all job basis is discussed, with several drawings showing plans for typical large jobs.

Following these somewhat general sections, major divisions of the book are presented—how to build forms for walls, columns, beams, floor and roof slabs, and stairs. Each of these structural members with its various ramifications receives intensive treatment. A glimpse of only a few of the topics considered gives an idea of the depth and scope of coverage: where and how to drive the nails; how to join one member to another; how to execute and brace corners; how to place external braces and shores; where to place ties and splices; how to deal with complicated beam-column and beam-girder intersections.

Brief discussion is presented on forms and falsework for shells, arches, tanks, silos, and hydroelectric installations. Several shorter sections deal with uses of standard plywood, and describe patented shores, plywood and metal panel systems, precast panels that serve as forms, and a variety of patented accessories. Interesting among the latter is a metal clamp for beam forms resembling in construction the column clamps familiar in the United States.

Various surface effects attainable with different forming materials and methods of construction are also discussed.

Parking garage spans street, uses thick hollow slab floors

Engineering News-Record, V. 165, No. 24, Dec. 15, 1960, pp. 42-47

Describes the construction of a parking garage spanning a downtown street in Omaha, Neb. Design called for 21,000 ft of octagonal paper tubes to form voids in the slabs. The seven level parking structure uses 26-in. slabs supported only by steel beams that run between the columns on the sidewalk lines. The use of lightweight aggregate concrete and the hollow section slabs made possible the long span. The street was open to traffic during construction.

New forming method for box-girder bridges

BILL ALLEN, *Contractors and Engineers*, V. 58, No. 3, Mar. 1961, pp. 16-18

A new method of forming box- girder bridges is discussed. The key to the forming system is in placing concrete for the bottom deck and the webs at the same time thus reducing the forming and placing time from 30 to 50 percent. The top deck forming is left in place. Only three separate concrete placements were necessary; one completed the lower deck and webs, two completed the upper deck.

Concrete construction: Guide to better field practice

Concrete Construction Publications, Inc., Elmhurst, Ill., 1960, 118 pp., $2

A compilation of articles that have appeared in *Concrete Construction*. This book is divided into the following sections: handling and placing, materials and testing, finishing, curing and protecting, crack prevention, floors and slabs, repairing, and design and specifications. Some of the specific topics covered are: consolidation of concrete, how to get quality concrete, types of cement, waterstops, decorative concrete, making slump tests, winter concreting, air entrainment, plastic cracking, driveway construction, concrete floors, thin bonded resurfacing, and many others.

A look ahead: Central mix equipment for concrete roads

G. MAXON, *Roads and Streets*, V. 103, No. 12, Dec. 1960, pp. 56-60, 64

Discusses mix equipment for road projects and how they can be improved. Mixing time, cement apparatus, mixers, aggregate batchers and bins, weighing equipment, power supply, hauling units, concrete spreaders, paving forms, coordination of equipment, retarders, and visible mixing are discussed.

Honeycomb of pipes damps echoes

JACOB J. CRESKOFF, *Engineering News-Record*, V. 166, No. 2, Jan. 12, 1961, pp. 39-40

Problem of damping echoes under a dome roof was solved using a honeycomb formed by 13,000 concrete pipe sections for Main Line Reform Temple, Wynnwood, Pa. The 70 ft diameter dome rises 20 ft, with a concrete compression ring at the apex framing an 8 ft diameter skylight opening over the center. The peripheral tension ring was post-tensioned to reduce possible shell cracking and to permit use of an economical, reinforced plastic roofing material.

Investigation of methods of finishing formed concrete surfaces

B. J. HOUSTON, *Technical Report* No. 6-559, U.S. Army Engineer Waterways Experiment Station, Corps of Engineers, Vicksburg, Miss., Dec. 1960, 22 pp., $0.50

Reviewed by MARY K. HURD

Three procedures for sack-rubbed finishing of concrete were selected from descriptions in literature of the American Concrete Institute, the Portland Cement Association, and the U. S. Bureau of Reclamation. Durability tests were made of panels finished by these different methods plus modifications of the PCA and USBR techniques.

Merely applying finish mortar with a brush or by burlap rubbing was found to give insufficient consolidation to bond finish grout tightly in the depressions. Superior durability was attained by rubbing a dry mix over the still-plastic mortar or by vigorously scouring the finish mortar with a wood float

after it had been applied to the concrete.

Evaluating the advantages and disadvantages of the three methods, a recommended finishing procedure was developed covering proportioning of mortar as well as the method of application and curing.

A new corrosion resistant concrete. Its use for the construction of pipes and sewers (in French)

G. Roederer, *Chimie and Industrie*, V. 84, No. 6, Dec. 1960, pp. 912-922
Battelle Technical Review V. 10. No. 4, Apr. 1961

The "Ocrate" process consists of treating the concrete, previously dried, by gaseous SiF_4. After this treatment, the concrete shows an increased resistance to corrosion by salts and acids and can be used in the construction of pipes for industrial waste waters and town sewage.

Sleeve method of splicing reinforcing bars

Owe Eriksson, *Ingeniören* (Copenhagen), International Edition, V. 4, No. 4, Dec. 1960, pp. 136-147

Describes the sleeve method of splicing deformed reinforcing bars, obtaining an axial transfer of force, with special regard to how the method can be adapted for joining precast concrete units. The splice method employs a cylindrical steel sleeve with inside deformations, and makes use of injected or preplaced high strength mortar grout to develop the necessary bond (shear) connection between bars and splice piece. Comparisons with conventional splice methods are given, together with a description of basic structural principles, applications, and cost estimate of the sleeve method. Laboratory tests of the splice are reported and recommendations are given with respect to the safe length of splice, thickness of mortar grout, design of splice cylinder, and other factors related to the practical use of the splice method.

Special equipment for concrete work on dam

Ralph Monson, *Contractors and Engineers*, V. 58, No. 2, Feb. 1961, pp. 14, 16-18

Describes placement of more than one million cu yd of concrete in the Wanapum Hydroelectric Development of the Columbia River near Vantage, Wash. The dam is a 8320 ft modified z-shaped reinforced concrete and earth fill structure. Revolving cranes on 85-ft gantries lift 6-ton cranes to the top of each 5-ft lift to aid in the removal of steel forms.

Deluxe traveling rigs help set and strip bridge deck forms

Roads and Streets, V. 103, No. 12, Dec. 1960, pp. 50-53, 101-102

Describes forming operations on the Woodrow Wilson bridge south of Washington, D.C. The forms for the cantilevered sidewalk were placed and stripped from an "offside" platform mounted on an 8-ton trailer, and hung over the side of the bridge. The trailer was counterweighted with 1800 lb of concrete and towed along the curb line as the work progressed. The formwork for the main slab was done from a cable-slung rig that could be pulled along the girders by hand. Additional forming operations are discussed.

Put science to work in space-age building

Joe Todaro, *Construction Equipment*, V. 23, No. 4, Apr. 1961, pp. 44-51

Describes unusual construction techniques applied to TWA's new terminal at Idlewild Airport, N.Y. The unique thin-shell roof called for intersecting-curve roof surfaces and massive-concrete supports. Four flying buttresses carry the 50,000 sq ft thin shell umbrella with no interior-column support. This necessitated precise forming and shoring, and control of machines, men, and materials. Around-the-clock concrete placing, and use of computers and lightweight concrete alleviated some of the many problems.

How improved paper curing helps get better concrete

J. M. CUMMINGS, *Roads and Streets*, V. 104, No. 4, Apr. 1961, pp. 68-68, 70

Compares curing with white waterproof paper to other methods. Charts compare relative moisture losses and maximum temperatures during cure with various curing compounds and slab coverings.

Dunlop factory sheds of reinforced concrete at Amiens (Usine Dunlop a Amiens sheds en béton armé)

M. HAHN, Bulletin, International Association for Shell Structures (Madrid), No. 1, 8 pp.

Describes the formwork and construction of 102 shells for the Dunlop factory in Amiens, France. The interior forms were simple making erection and stripping easy. The forms folded down and when the scaffolding was lowered the whole assembly was moved to the next bay. One shell was completed each working day.

New tunnels near Potters Bar in the eastern region of British Railways

ALEXANDER K. TERRIS and HORACE D. MORGAN, *Proceedings*, The Institution of Civil Engineers, London, V. 18, Apr. 1961, pp. 289-304
AUTHORS' SUMMARY

Describes the method of constructing three shield-driven concrete-lined tunnels through London clay, on a novel principle. This involves the use of dry joints between precast concrete blocks with no break of joint, continuity in alignment and radially being obtained with only relatively small tongues and recesses in the sides and ends of the blocks.

The tunnels were driven alongside three older ones of conventional design, built in 1849-1850, which had to be maintained in constant use throughout the duration of the works. The new tunnels are of shallow depth and the longest one is provided with two ventilating shafts and a throat to permit the subsequent construction of a third should the need arise.

All the concrete segmental blocks were manufactured in a special factory located over the top of one of the tunnels. The nature of the work required the segments to be made within fine limits of dimension and the quality of the finished product was checked, *inter alia*, by ultrasonic means. Two kinds of cement were used in the tunnel lining.

Intricate work for a modern Zoo

Contractors and Engineers, V. 58, No. 5, May 1961, pp. 51-52, 57

Emphasizes construction difficulties in building Milwaukee's new zoo. Perhaps the most difficult part was transforming concrete walls into stone cliffs. First, ½ in. steel anchor rods are set in the joints of the outside wall panels. In some cases, one end of the form tie is left in place to serve as an anchor. These "protruding" anchors serve as welding points for a framework of reinforcing bars. The bars are bent to conform to the irregular shape of the desired formation. Dimensions for the shape are scaled from a model. Completion of the zoo is scheduled for 1964.

Field methods in concrete construction—Parts 1 and 2

A. DEGROOT, International Textbook Co., Scranton, Pa., 1960, Part 1—98 pp., Part 2—104 pp.

A good introduction for the student to the equipment and techniques used in concrete construction. Of necessity, much of the material is handled briefly since these two small textbooks span the subject of concrete construction from the preparation of aggregates to such specialized methods as lift-slab construction.

Part 1 covers raw materials (preparation of aggregates, handling cement, measuring materials); manufacture of concrete (principles of mixing, mixers, plants), handling mixed concrete (conveying, placing), and treating concrete in place (consolidation, finishing, curing, sawing).

Part 2 devotes major attention to prestressed concrete (materials, equipment, forms, concreting), lift-slab con-

struction, tilt-up construction (construction of panels, tilting, construction of columns) and organization for concrete construction (contractor's forces, engineer's forces, inspection).

Place concrete by air in tight quarters

Contractors and Engineers, V. 58, No. 2, Feb. 1961, pp. 28-31

Describes pneumatic concrete placement for prestressed tanks of a water-treatment plant in Santa Cruz, Calif. The structures were built on benches excavated out of a steep hillside that ruled out the crane-and-bucket method. The mix was carried through forms by a pipe that rises in the center of the tank to a swivel elbow and is carried out to the forms on a truss.

Dams

Concreting techniques at Glen Canyon Dam

JOSEPH PERAINO, *Civil Engineering*, V. 31, No. 7, July 1961, pp. 50-53

The story of concreting operations for Glen Canyon Dam on the Colorado River is a story of automation. Some 5.2 million cu yd of concrete is to be placed in the dam and powerhouse over a 2500 day construction period. The contractor devised a completely automated plant to mix, transfer, and place concrete. Aggregate plant, batch plant, and placing of concrete is described.

Dokan dam in Irag (Le barrage de Dokan en Irak)

M. P. FOUILLADE, *Annales*, L'Institut Technique du Bâtiment et de Travaux Publics (Paris), V. 13, No. 154, Oct. 1960, pp. 1139-1156

General description of the 380 ft high concrete gravity arch Dokan dam in Iraq designed for flood control on the Lesser Zab River and irrigation of 2700 sq miles. A total of 686,000 cu yd of concrete made of cooled materials was also refrigerated in the dam after placement. Floods and political difficulties added to the problems of the actual construction.

Meffrouch Barrage

The Engineer (London), V. 211, No. 5483, Feb. 24, 1961, pp. 313-314

Reviewed by ARON L. MIRSKY

Barrage, in Western Algeria, was intended to be a multiple-vault dam of ordinary concrete construction, but local conditions, including lack of construction plant and qualified workmen, foroed change in plans. Most of structure above ground, including all 17 vaults and their abutments, is of precast concrete sections, while the foundations and the hillside abutments are of mass concrete. Advantages included minimum technical staff and maximum use of local labor. High precision was required in the fabrication of the precast components, since no centering was used.

Methods and materials for the study of the weight stresses in dams by means of models

J. L. SERAFIM and J. POOLE DA COSTA, *RILEM Bulletin* (Paris), No. 10, New Series, Mar. 1961, pp. 49-57

AUTHOR'S SUMMARY

The determination of the stresses due to the weight of the body in small models of dams is important, particularly in arch dams where the double curvature is marked, in gravity dams, and in buttress dams. The experimental studies made it possible to determine the influence of the order of construction of the dam on the state of stress arising from the dead weight of the structure.

The methods used for the experimental determination of stresses due to the weight of the body are briefly presented and the methods of inversion, immersion of the model in dense liquids, application of concentrated loads, and construction in stages are analyzed in great detail with various cases studied in the LNEC being quoted. Finally because of the difficulty of each method as regards the choice of satisfactory materials, the advantages of the method of construction in stages are stressed, as this method reduces the problem of the choice of materials to a minimum.

Stress analysis and special problems of prestressed dams

O. C. Zienkiewicz and R. W. Gerstner, *Proceedings*, ASCE, V. 87, No. PO1, Jan. 1961. pp. 7-43

Authors' Summary

The development and use of cable methods of prestressing as applied to dams is presented. Some techniques currently in use are examined.

The problem of stress distribution within the foundation and near the base of the dams is found by superposition of known elasticity solutions with a finite difference correction. The results of the analysis are discussed and some general conclusions arrived at. A suggested method for a simplified analysis is presented.

Design

Design curves for singly reinforced slabs in liquid-containing structures

J. D. Davies, *Civil Engineering and Public Works Review* (London), V. 56, No. 656, Mar. 1961, pp. 331-332

Design tables and charts are presented based on "Design of Slabs in Liquid Containing Structures," by C. E. Reynolds in *Concrete and Constructional Engineering* (London), Feb. 1940. A numerical example is also included.

Cracking theories and equations

English translation of *Bulletin D' Information* No. 12, Comité European du Béton, Paris, Feb. 1959, 40 pp. (Limited supply of single copies available from ACI headquarters)

A translation from the French of *Bulletin* No. 12, CEB. The general theory for circular bending and for direct tensile force is discussed and equations are presented for cracking of beams subjected to bending and cracking of members in tension, taking into consideration the distance between cracks and the width of cracks. Various existing theories are compared, including Brice's theory, statistical researches by Efsen and Krenchel, Haas' researches, Rüsch's theory, Saliger's theory, and the theory of Wästlund, Jonsson, and Osterman. Research conducted at the Building laboratories and at the Public Works is described briefly. The last chapter of the report is devoted to simplified practical formulas as proposed by the various researchers for predicting the maximum width of cracks that will occur under a given loading. It is shown that the formulas by the various authors can be written in the same theoretical form.

Hanging roof in Bahia Blanca

Jose N. Distefano and Ricardo A. Arrigoni, *Bulletin*, International Association for Shell Structures (Madrid), No. 5, 1961, 16 pp.

Describes the important parts and the static behavior of a hanging roof to cover a basketball stadium of 40 x 50 m with perimetral tribunes of reinforced concrete.

The roof is formed by light concrete tiles which lie on hanging steel cables. The cables are anchored in two concrete arches.

Shrinkage, creep, and temperature effects which modify the stresses are also discussed.

Cylindrical concrete tanks on elastic foundations

J. D. Davies, *Civil Engineering and Public Works Review* (London), V. 56, No. 654, Jan. 1961, pp. 61-64

Author's Summary

Paper describes a method of determining the bending moments developed in a cylindrical concrete tank supported on a foundation in which the reaction pressure at a point is proportional to the deflection at that point. Linear elastic behavior of the tank is assumed and the analysis of the floor slab is simplified by using an approximation which is adequate for design purposes.

Most of the information is provided in a graphical form and the use of these charts in conjunction with the principle of superposition provides a rapid means of calculating the bending moments set up in the tank under specified support conditions.

The application of the method is demonstrated by an example in which comparisons are made between the

structural behavior of a tank supported on (1) an elastic foundation; (2) a foundation giving uniform reaction pressure; and (3) an edge ring beam.

Free-standing stair

PAUL ROGERS, *Progressive Architecture*, V. 42, No. 2, Feb. 1961, pp. 172-173

Analysis of design of a challenging free-standing stair which was recently constructed in St. Louis, Mo. Although similar stairs have been designed by empirical methods, this structural solution follows a precise mathematical analysis developed in Germany.

Philosophy of direct design

L. N. PRISMALL, *Journal*, South African Prestressed Concrete Development Group, (Johannesburg), Feb. 1960, pp. 1-20
Reviewed by D. G. NORMAN

Part of the proceedings of the group's first international congress, the paper is written in a humorous vein by a talented engineer in private structural design practice. Deals with short cuts in design brought about not by empirical charts, but by a revision of the fundamental approaches to design. Author also uses successive approximations in proportioning of beam cross sections.

Recent developments in structural concrete

A. W. HILL, *Reinforced Concrete Review* (London), V. 5, No. 2, June 1959, pp. 65-97
Reviewed by WILLIAM R. LORMAN

Postwar developments, in the design and construction of conventionally reinforced and prestressed concrete, are treated under five main headings: (a) materials (concrete and reinforcement); (b) constructional techniques (precast, composite, in situ, lift slab); (c) structural forms (shell roof, multistory, factories, bridges, automobile parking structures, flat slabs, folded plates); (d) surface finishes (texture mainly); and (e) research. This presentation apparently was made in preprint form, rather than oral, for the author's supplementary remarks constitute the first six pages of the discussion. The ensuing comments by eight discoursers cover features such as the practicalities of epoxy resins (as jointing material) and concrete mixtures having unusually high aggregate-cement ratios. The paper proper contains eight photographs which illustrate the author's main points.

Engineering design in brickwork

B. H. FALCONER, *New Zealand Concrete Construction* (Wellington), V. 5, No. 5, June 12, 1961, pp. 79-86

The strength of brick and brickwork is reviewed, with reference to provisions of various codes. Provisions of codes giving maximum allowed ratios of length and height of panels to thickness are noted. The section "Design Analysis and Working Stresses" from the New Zealand NZSS 95 Part X:1959 is quoted. The basis of engineering design of masonry in conformity with the latter code is indicated, particularly for walls of unreinforced masonry including cavity walls.

The structural design of reinforced concrete biological shields

P. A. BADLAND, *Reinforced Concrete Review* (London), V. 4, N. 8, Dec. 1957, pp. 469-495
Reviewed by WILLIAM R. LORMAN

Concerned mainly with the complexities of concrete shielding for nuclear reactor plants. Necessary properties of the shield, thermal stresses, density, differential expansions, cracks, joints, steel fracture, and construction are lucidly but concisely described. The suggested method of design assumes that Poisson's ratio is zero. The section on thermal stresses receives special attention. In addition to a brief statement regarding the advantages of concrete creep, relative to thermal stresses, the author points out the necessity for elastic modulus and concrete creep experiments at higher than normal temperatures. Contains four photographs and five diagrams to illustrate the design method. The ensuing discussion by ten individuals ranges from concrete mix proportioning to strain distribution assumptions.

On the analysis of shear in highway bridge decks (Sur le calcul au cisaillement des planchers des ponts-routes)

SERGE WOINOWSKI-KRIEGER, *L'Ingénieur* (Montreal), V. 46, No. 183, Autumn (Sept.) pp. 21-25

Reviewed by ARON L. MIRSKY

Expressions are given for several types of edge conditions and two types of loading (point loading, uniformly distributed loading). The slab of a T-beam bridge (beams 10 ft c-c) under H20-44 truck loading is investigated as a numerical example.

Reactor shielding

KURT BILLIG, *Civil Engineering and Public Works Review* (London), V. 55, No. 650, Sept. 1960, pp. 1143-1145; No. 651, Oct. 1960, pp. 1295-1297; No. 652, Nov. 1960, pp. 1491-1493

AUTHOR'S SUMMARY

The radiation emitted from the core of a nuclear reactor consists of fast neutrons, thermal neutrons and gamma rays. For health reasons, the amount of radiation that the human body is permitted to receive is limited. Biological rays. For health reasons, the amount of radiation that the human body is permitted to receive is limited. Biological shields are therefore constructed around the reactor and its ancillary equipment to attenuate radiation to a permissible level. Concrete is by far the most widely used material for this purpose. In view of the high operating temperatures of nuclear reactors and the considerable heat produced within the shield by the absorption of neutrons, thermal stresses become the primary consideration in the design of such shields. The thermal properties of concrete and thermal stresses are therefore discussed here in detail, as well as thermal shields which are often employed to reduce the heat load on the biological shield. Heat generation and temperature distribution in the shield can be relatively quickly determined by the method of approximation described in the paper. Other factors, important to the design of reactor shields, also discussed in the report, are the question of shield reinforcement, the resistance of concrete to radiation, the effects of irradiation on steel, the use of heavyweight concrete, and the determination of the shield thickness. A description of research and development work in this field concludes the report.

The simplified design of concrete hyperbolic paraboloid roofs

COLIN ROSS, *Civil Engineering and Public Works Review* (London), V. 56, No. 654, Jan. 1961, pp. 65-66

Describes the design of two concrete paraboloid roofs constructed in Jamaica. Structural details and illustrations of the two roofs are also included.

Using the ultimate strength theory in design eccentrically-loaded rectangular concrete columns

B. GOSCHY, *Civil Engineering* (London), V. 56, No. 656, Mar. 1961, pp. 348-350

The ultimate strength design procedure of investigating eccentrically-loaded short and long columns presented herein enables one to solve the problem with ease and rapidity. Although this investigation is applied to the basic case of hinged rectangular columns, other special cases may be easily deduced from it. If permissible direct loads are required, the computed ultimate values have to be divided by a safety factor. The theory has been verified by extensive test data.

Practical shell statics—V. 1: Shells of rotation (Praktische Schalenstatik —V. 1: Die Rotationsschalen)

JOACHIM BORN, Wilhelm Ernst and Sohn, Berlin, 1960, 220 pp., 60 DM

The membrane theory is described and methods of calculation are derived for shells of cylindrical, conical, and spherical shapes. The mathematics of this type of shell structure are dealt with exhaustively. A graphical method is described which is suitable for any shell the shape of which is a surface of revolution subjected to vertical, horizontal, and wind loading. Other chapters cover deformations arising from

membrane action, bending of shells which involves hyperbolic differential equations, effect on a circular wall of a nonuniform rise of temperature, and design of ring beams in structures composed of spherical, cylindrical, or conical elements. Numerical examples are given for cylindrical and conical tanks.

Farm building design

L. W. NEUBAUER and HARRY B. WALKER, Prentice-Hall Inc. Englewood Cliffs, N. J., 1961, 600 pp., $8.50

Covers functional and analytical design of farm buildings, including farm houses, barns, sheds, shades, storage structures, grain bins, silos, animal shelters, water supply facilities, and septic tanks. The application of a variety of materials, including concrete, are discussed. For the experienced practitioner, the inclusion of elementary information at the expense of more advanced details on the application of concrete is to be regretted. Precast and tilt-up construction, for example, are not included.

Engine foundations

D. A. NESTERENKO, *Pipeline Engineer*, V. 33, No. 5 and 6, May and June 1961, pp. D28-D30 and D40, D41, D46, and D50

Presents information needed for design and construction of reinforced concrete equipment foundations of the block type. Block foundations are used to support hammers, diesel engines, air compressors, blowers and fans, etc.

Generally, three steps are necessary in the design of engine foundations: procuring general information, dynamic investigation, and structural design. General information includes soil investigations and basic engine data. The dynamic investigation is discussed in considerable detail and material is included on vibrations, computing inertia moments, and evaluation of the dynamic properties of the soil supporting the engine foundations. Structural aspects are covered. Fifteen general design rules are listed that are applicable for many types of foundations.

Reinforced concrete chimneys

C. PERCY TAYLOR and LESLIE TURNER, Concrete Publications Ltd., London, 2nd Edition, 1960, 85 pp., $2.80

This edition has been revised to include up-to-date methods of design and examples of more recent chimneys than those illustrated in the 1940 edition.

An improved method of analyzing annular sections subjected to bending and direct thrust replaces the more complex method given in the previous edition. Some of the design data, such as the combination of thermal and structural stresses, are also applicable to the design of structures other than chimneys, namely reinforced concrete tanks and biological shields. A new chapter is included and deals with the deflection and sway of tall chimneys due to the effects of wind. As in the previous edition, an example of the complete design of a chimney is given.

Some recent developments in the design of reinforced concrete shell roofs

J. D. BENNETT, *Reinforced Concrete Review* (London), V. 5, No. 1, Mar., 1959, pp. 23-62
Reviewed by WILLIAM R. LORMAN

Presentation of a wholly empirical design method based on an analysis of results of 227 complete calculations for barrel vault roofs. The method involves only the use of graphs and a slide rule, and is applicable to the following symmetrical cylindrical shells: (a) feather-edged barrels, (b) internal barrels with valley beams, and (c) barrels with vertically supported edge beams. These types of barrels, and relevant curves showing typical distribution of the shell forces, comprise the first two figures. A detailed description of the design method includes 16 additional figures. The author claims that the method is useful for those engineers who do not possess the specialist's knowledge in this area; specifically, " . . . an exact elastic analysis of a barrel is a waste of time if other simpler methods are available." The paper, which lists two references, is followed by a seven-man discussion which covers

the use of the graphs, measured versus calculated deflections, and general comments relative to cracking of shell roofs.

Stresses in shells

WILHELM FLUGGE, Springer-Verlag, Berlin, 1960, 500 pp., 58.80 DM

After a preliminary chapter on fundamentals in this English language book, three chapters deal with the membrane theory, i.e., the theory of shells whose rigidity against bending may be neglected. Advantages as well as shortcomings of this theory are examined closely. Two chapters are then devoted to the bending theory of circular cylindrical shells and the general shell of revolution. In the chapter on stability of shells a number of selected stability problems are explored as representative of the present state of knowledge.

The book is aimed at the graduate level and especially to design engineers and research workers.

Eccentric compression

English translation of *Bulletin D'Information* No. 15, Comité European du Béton, Paris, Mar. 1959, 21 pp. (Limited supply of single copies available from ACI headquarters)

Report of Committee 3 of the Comité Europeen du Béton by Rüsch on eccentric compression. The appendix on compression reinforcement is authored by Aas-Jakobsen. The first report of the ACI-CEB joint committee, already printed in English in the original *Bulletin d'Information* No. 15 has not been included.

It has been established that there is a continuous transition between the two extremes represented by simple bending and axial compression. Eccentric compression is therefore merely a special case of the general problem of the strength of members subjected to uniaxial stress. The result is that, for all conditions of stress under consideration, the strength can immediately be calculated as soon as a satisfactory assumption can be made for the fundamental relation between stress and strain. This investigation is concerned with what form this fundamental relation should take and the problem of the influence of the duration of loading.

A general theory of uniaxial stress is outlined. The importance of the load duration in design is discussed. Use of available knowledge of structural behavior for design formulas is discussed and four approaches are suggested.

Tables for combined bending and axial load on reinforced concrete columns — Cracked sections (Output listing of IBM tape)

SABY ELIACHAR, Civil Engineer, Daly City, Calif., 1960, 333 pp., $25

AUTHOR'S SUMMARY

The tables contained in this publication, which is a listing of the output tape of an IBM 704 computer, treat the time-consuming case of design of reinforced concrete columns by "cracked sections." The analysis of columns by cracked sections is required by ACI 318-56 whenever the ratio of eccentricity to column thickness exceeds 2/3. For example, in the tables, columns whose depth (thickness) is 12, 18, 24, and 30 in. of which 2/3 thereof is an integer, such integer was used as the starting value for the eccentricity. Other columns start with an eccentricity the next higher integer than 2/3 the column depth. The computational procedure follows the well established one of solving the cubic equation for KT, namely the distance of the "crack line" from the compressive face of the column. The provisions of Section 1109 (d) of the ACI Code are followed, and the value for the modular ratio for compressive reinforcement is taken as twice the value for tensile reinforcement.

The tables list 999 combinations of column dimensions and reinforcing, ranging from 12 to 30 in., and thus cover a great range of rectangular and square columns (almost 30,000 designs). The only limitation in the tables is that the steel be arranged symmetrically with respect of the axis of bending,

and that it be located 2½ in. from the face of the column. For each column 15 values of eccentricity are tabulated, starting from the minimum eccentricity for which cracked section should be considered. For each such eccentricity, the following is tabulated both for intermediate grade steel and hard grade steel: (1) the distance KT, (2) the allowable load on the column, (3) the computed tensile and compressive stresses in the steel reinforcement, (4) the compressive stress in the concrete.

Comparison of specifications and recommendations for design of pretensioned concrete members

A. WILLIAMS, *Journal*, South African Prestressed Concrete Development Group (Johannesburg), Apr. 1960, pp. 1-38
Reviewed by D. G. NORMAN

Papers were read at a prestress symposium in Rhodesia, and compare American, Australian, British, and German codes for design of prestressed and normally reinforced concrete. Materials, stresses, losses, and shear are discussed.

Composite construction and ultimate load design are treated in detail, and results of tests on composite units are given.

From this study a Rhodesian specification was developed, and the paper is useful since it encompasses various approaches.

Influence of the shear and longitudinal forces and the rotational inertia on the vibration of foundation frameworks (in Polish)

EDWARD KRYNICKI, *Archiwum Inzynierii Ladowej* (Warsaw), V. 6, No. 3, 1960, pp. 271-305
Reviewed by S. GALEZEWSKI

An interesting paper dealing with the foundation vibrations. Equations for the computation of symmetric and antisymmetric, free and forced vibration, of statically indeterminate portal frames are derived. Tables are shown enabling the application of the equations derived to engineering problems.

Three problems of actual foundation frames are completely worked out, and the equation derived and the tables prepared enable rational and much more accurate computation of the dimensions of frameworks with large cross sections and working under dynamic loads.

Reinforced concrete — Flexure and bending compression of circular and octagonal cross sections (in Portuguese)

F. PEREIRA SEQUEIRA, *Bulletin*, Direcçao dos Serviços de Obras Públicas e Transportes (Luanda, Angola), No. 8, Oct.-Dec. 1960, pp. 183-205
AUTHOR'S SUMMARY

A review of design methods. First, the author presents the calculation for circular cross sections with graphical determination of the location of the neutral axis and the moment of inertia from the transformed cross section about the same axis. Guerrin's method is discussed and examples are given of the use of Hahn's diagram. Flexure of the octagonal cross section as treated by Mörsh is described and examples given. The maximum unit shearing stress formulas are analyzed and example problems worked.

Industrial architecture: An analysis of international building practice

JAMES F. MUNCE, F. W. Dodge Corp., New York, 1960, 232 pp. $14.75

A comprehensive and stimulating review of the basic principles and newest developments in factory design in the United States, Great Britain, and Germany. The book covers the history of industrial building design, examines its present state, and emphasizes changing trends and their implications for the future. The aspects of industrial architecture included are: functions and needs, structural elements, services, physical appearance, and location. Other chapters discuss requirements of heavy industry, light industry, the process industries, and the utilities industries. Sections are devoted to structures, materials, and design techniques including types of roof structures, external cladding, and flooring.

The design and construction of shell roofed buildings at Moreton, Cheshire

J. E. C. FAREBROTHER, *Reinforced Concrete Review* (London), V. 5, No. 3, Sept. 1959
Reviewed by WILLIAM R. LORMAN

A narrative and pictorial description of a biscuit (cookie) factory comprised of about eight major buildings. Furnishes specific information regarding the design and construction of the building used in fabricating cardboard boxes (containers). The so-called card box building is two stories high, 350 ft long, 200 ft wide, and consists of 50 ft span shells; the main floor, 12 in thick slab supported on columns, is designed to carry loads of 336 psf, and the shell-type piles are designed to support 80 tons each. A total of 10,000 cu yd of concrete was produced by a central mixing plant using a 21-cu ft mixer and Portasilo bulk cement storage containers. The paper contains three diagrams and ten excellent photographic illustrations of various construction phases (including special formwork). No references nor discussion. The engineer engaged in concrete shell-roof construction will find this paper of considerable interest.

Initial values problem for differential equations system applied to the analysis of helicoidal girders (in Turkish)

V. CINEMRE, Dissertation, Technical University of Istanbul, 1960
Reviewed by H. S. GEDIZLI

The analysis of continuous helicoidal girders by the initial values method for the differential equations system is treated. The Fuchssteiner-Menn's differential equations system for the equilibrium of a helicoidal girder is solved and for a continuous helicoidal girder with many spans the number of redundant quantities is reduced to six (similar to the analysis of continuous straight beams with the three-moment equation).

Initial values of a helicoidal or circular girder according to the cartesian or cylindrical coordinates and loaded horizontally or vertically are given. The initial value coefficients for a thin rectangular helicoidal girder are also numerically computed.

As numerical examples: (a) one span helicoidal girder fixed at both ends; (b) one span helicoidal girder fixed at one end and freely supported at other end; (c) two spans helicoidal girder; (d) three spans helicoidal girder; and (e) temperature effects, are analyzed.

Thus, the analysis of any continuous helicoidal or circular girder under concentrated or uniformly distributed loads is reduced to (1) taking the appropriate initial values from the tables and (2) solving the six linear equations with six unknowns.

Calculation of the ultimate flexural strength of reinforced concrete subjected to simple or compound flexure — Statistical comparison of various theories with the published results of experimental researches

C. MASSONET and P. MORNARDT, *Preliminary Publication*, Sixth Congress Stockholm, Association for Bridge and Structural Engineering (Zurich) 1960, 22 pp. (English translation No. 61-10138 available from SLA Translations Center; microfilm $2.70, photocopy $4.80)
TECHNICAL TRANSLATIONS
V. 5, No. 6, Mar. 28, 1961

Four methods for calculating the breaking strength of reinforced concrete members subjected to simple or compound bending have been compared with the test results published in the literature and comprising a total of 1553 usable tests. These tests relate to members having a rectangular, T, or inverted-T section, with a single or twin reinforcement made of mild steel or high tension steel. The four methods of calculation are based on the two equilibrium equations, the law of the conservation of plane sections and the properties of the concrete and steels employed, as defined by the European Committee on Concrete. They differ solely on the shape adopted of the stress diagram of the concrete under compression; the forms studied were: the parabola, the rectangle, the tri-

angle, and the distribution law suggested by Hognestad, Hanson, and McHenry. The calculations were carried out using a single program on an IBM 650 computer. For the various types of beam that were tested the publication gives the mean values and the degree of scatter of the ratio r (observed moment or rupture/calculated moment of rupture).

Frame analysis

A. S. HALL and R. W. WOODHEAD, John Wiley & Sons, Inc., New York, 1961, 248 pp., $8.50

A mathematical analysis of structural frames giving principles of flexibility analysis and stiffness analysis. Several well-known methods are appraised briefly to show that all other techniques are variants of these two methods of analysis. Matrix algebra is used extensively and an appendix explains those parts used in the book.

Simple reinforced concrete design tables in metric units

Concrete Association of India, Bombay, 1960, 28 pp., 50 nP

In 1956 the government of India was authorized to introduce the metric system as the only system of weights and measures in India within 10 years. Since then much has been done to implement the change. In keeping with this trend all new codes and specifications of the Indian Standards Institution are now being formulated on the basis of metric units and soon all measurements and specifications pertaining to public works will also be required to conform to the metric system. A conference in 1959 trusted the preparation of reinforced concrete design tables and metric units to the Concrete Association of India. Scope of this publication has been restricted to loading standards for buildings and to the design of solid slabs, beams, columns, and footings. As far as possible only design tables have been included and the text is confined to an explanation of their use in practice. The tables and loading standards are based on two Indian Standards—(1) Code of Practice for plain and reinforced concrete for general building construction (No. 456-1957), and (2) Code of practice for structural safety of buildings (No. 875-1957).

Materials

Sodium silicate for concrete protection

Building Construction, Feb. 1961, pp. 56-57

Answers question of where to specify and how to apply new silicate sealer for interior concrete subject to oil, acid, vehicle abrasion, etc. Cost is under 15 cents per 100 sq ft in three-coat application.

The selection of natural aggregates for various types of concrete work

J. D. MCINTOSH, *Reinforced Concrete Review* (London), V. 4, No. 5, Mar. 1957, pp. 281-305
Reviewed by WILLIAM R. LORMAN

Describes various aggregate properties and the influences thereof on different types of concrete. In the section dealing with aggregate production the author stresses the relationship between types and qualities of aggregates; he shows how certain characteristics may be controlled if careful processing of local source materials is observed. Contains 14 references but no discussion.

High temperature compressive strength of concrete

HANS LEHMANN and G. MALZIG, *Tonindustrie-Zeitung und Keramische Rundschav* (Berlin), V. 84, No. 17, 1960, pp. 414-417
CERAMIC ABSTRACTS
V. 44, No. 6, June 1961

The strength of concretes containing portland or blast furnace slag cement and graywacke or blast furnace slag increased with temperature to a maximum (approximately 150 percent of the cold strength) at 300 C and then decreased again to the initial value. Graywacke is more suitable than blast-furnace slag, and limestone is quite unsuitable. Portland and blast furnace slag cement are equally suitable. The maximum temperature, not the duration of setting, is the important factor.

Concrete admixtures

M. SANDALL, *Reinforced Concrete Review* (London), V. 4, No. 9, Mar. 1958, pp. 559-568
Reviewed by WILLIAM R. LORMAN

Presents current thoughts relative to: (a) accelerators and retarders; (b) integral waterproofing agents; (c) cement-emulsion mixtures; (d) air-entraining practices; (e) plasticizers; (f) wetting agents; and (g) foaming agents for cellular concrete. The latter half of this paper is concerned with properties (strength, density, thermal conductivity, shrinkage, swelling, permeability) and principal uses of lightweight concrete. A one half page long section on concrete surface treatments (liquid and gas) concludes the presentation. No illustrations nor references nor discussion. A good thumbnail sketch of present trends.

Grouted concrete construction

S. CHAMPION, *Reinforced Concrete Review* (London), V. 4, No. 10, June 1958, pp. 569-608
Reviewed by WILLIAM R. LORMAN

Deals with all aspects of prepacked concrete made in accordance with the Colcrete process in which the colloidal grout is known as Colgrout. This treatise covers not only grout characteristics and methods of mixing but points out the advantages and applications of grouted concrete (i.e., prepacked concrete as commonly termed in the United States). Good examples of this type of concrete construction are presented with sufficient detail to make the paper interesting from beginning to end; the author gives examples of mass concrete, piles and caissons, heavily reinforced foundations, highway paving, underwater construction, and miscellaneous uses ranging from prestressed concrete to pneumatically applied grout. The too brief appendix pertains to testing methods peculiar to this type of concrete. In the discussion ten authorities cover topics such as heat of cement hydration, piles, grout consistency, aggregate voids, grout penetration, and differences between the Colcrete (British) and the Intrusion Prepakt (United States) systems.

Suitability of cement mortars with polyvinyl acetate admixture for the repair of concrete surfacings (in German)

W. ALBRECHT, *Strassen- u. Tiefbau* (Heidelberg), V. 14, No. 7, 1960, pp. 516, 518, 520, 522-531
ROAD ABSTRACTS
V. 28, No. 2, Feb.

Details are given of laboratory research at the Otto Graf Institute, Stuttgart, and full-scale tests on the motorway between Darmstadt and Kassel.

Lightweight concrete (Leichtbeton)

G. ROTHFUCHS, Bauverlag, Wiesbaden, 1959, 156 pp.
Reviewed by H. H. WERNER

This book on lightweight precast concrete products is a well-described and illustrated summary of the state of this industry in Germany. Development, materials, fabrication, properties, design, and standards are treated methodically. Practical applications with many details, up to lightweight bearing block for 5-story structures, are shown from this rapidly expanding industry.

Automation in aggregate production (in Russian)

I. I. LIFEROV and I. T. TESEL'KO, *Mekhanizatsiya Stroitel* (Moscow), V. 17, No. 12, 1960, pp. 8-11
ROAD ABSTRACTS
V. 28, No. 1, Jan. 1961

Information is provided on the use of remote control in crushing and screening operations at the Tsarekonstantinovsk aggregate plant in the USSR. The system (illustrated by a diagram) is controlled from a central panel, using weak-current, low-voltage circuits on six independent ducts, which direct the working of the primary and secondary crushing plant, the screening plant, the loading and reject bunkers, and the inter-plant conveyor. After preliminary tests, the plan was in production for over a year without interruptions or breakdowns. The aggregate sizes produced were 25 to 70 mm and 7 to 25 mm.

Corrosion of iron and steel embedded in concrete (in Swedish)

TENHO SNECK, *Nordisk Betong*, (Stockholm), V. 1, No. 1, 1961, pp. 1-28
Reviewed by MARGARET CORBIN

The alkaline nature of portland cement results in the formation of a passivating film on steel embedded in concrete. This places the steel into the passivation area of the Pourbaix diagram. The film may be inhibited by: carbonation of calcium hydroxide, porosity and poor compaction of concrete, the nature of the contact between steel and cement in the concrete, thickness of the concrete, cracks, and cement content. The corrosion mechanism is an electro-chemical process of contact corrosion, with oxygen diffusion or the destructive agent. Several practical cases of corrosion, particularly in pipes, are examined. Suggested protection treatments are reviewed. There are 29 references.

Application of concrete admixtures in highway construction: Results of a survey by the committee on chemical additions and admixtures for concrete

COMMITTEE ON CHEMICAL ADDITIONS AND ADMIXTURES FOR CONCRETE, *Highway Research Abstracts*, V. 31, No. 5, May 1961, pp. 25-30

Reports on results obtained from a questionnaire circulated by this Highway Research Board committee to the highway departments of the 50 states and the District of Columbia. Objectives of the study were to learn the classes of admixtures being used, the purposes for which they are used, and problems arising from their use. Admixtures were listed by categories and respondents were asked to indicate (a) what types are permitted in pavement and structures, (b) how extensively the particular type is used, (c) what specifications, if any, are used to control them, and (d) other pertinent information. The listing which corresponded roughly to that in the ACI Committee 212 report "Admixtures for Concrete" covered accelerators, air-entraining agents, alkali-aggregate reaction controls, cementitious materials, pozzolans, waterproofers, water-reducing agents, workability agents, and others. The extent of use of each of the above are summarized and a table provided listing use by states. A brief summary is included on what specifications are used by highway departments to control the quality and performance of admixtures. Application of admixtures to specific problems and problems of misuse of admixtures are also summarized.

The committee concludes that concrete admixtures are extensively used in highway pavements and structures. Air-entrainment is employed in virtually every state including those where weathering would not be considered severe. Calcium chloride is widely used to increase the rate of strength development mostly to reduce the time of needed protection in cold weather. Water reducing agents and retarders have achieved considerable stature and are coming into widespread use. Admixtures are seldom used specifically for the enhancement of workability, although that result may derive as a by-product from water reducers or air-entraining agents. Only limited application is made of admixtures for control of alkali-aggregate reaction, for waterproofing, or because of cementitious or pozzolanic properties.

The few available specifications for admixtures, such as those of ASTM and AASHO for air-entraining agents, calcium chloride, and fly ash, are generally accepted and used by highway departments. There are only a few cases of states having their own specifications. It is noted that a fairly common practice is to leave acceptance of admixtures to the project engineer or include provisions in the individual contracts for a specific product or its "equal." It was apparent to the committee that more widely available specifications of test methods for admixtures particularly water reducers and retarders are needed.

Mineral use guide

R. H. S. Robertson, Cleaver-Hume Press, Ltd., London, 1960, 21s

Consists of a brief introduction, 32 diagrams (Robertson's Spiders' Webs) on uses of rocks and minerals, and a glossary of terms. In general, the innermost part of the diagram shows the common unit processes used to prepare the mineral or rock for commercial use, the second ring gives the chief physical properties, while the outer lists the applications.

Automation spurs sampler design

John A. Mitchell, *Rock Products*, V. 64, No. 6, June 1961, pp. 109-110, 112, 114, 116, 118, 136

A roundup of the automatic samplers now being used in the cement industry. The features desired in the ideal automatic sampler are described. To get a picture of the automatic sampling methods, seven types of samplers as used in 13 plants are described: screw-type sampler, cup-type sampler, helical tube-type sampler, cutter type sampler, scoop type sampler, the Vezin sampler, and raised slot sampler.

Prepacked concrete and activated mortars (Béton de blocage et mortiers activés)

J. Chefdeville, *Annales*, Institut Technique du Bâtiment et des Travaux Publics (Paris), No. 144, Dec. 1959, pp. 1313-1332

Reviewed by Henri Perrin

Prepacked aggregate concrete is made of a skeleton of pebbles whose voids are filled by injected mortar. Various processes are used to activate the mortar: mechanical by mixing into special rotating mills; chemical with adjunction of additives; or by combining both mechanical and chemical processes.

Laboratory tests to determine the criteria of injectability are reported. Further, comparative tests with normal concretes make it possible to ascertain the mechanical properties of prepacked concrete.

The results have shown that prepacked concrete hardens quicker than normal concrete. The compressive strengths at 28 days and 7 years are similar but the tensile strengths are lower. Instantaneous elasticity modulus is practically the same; shrinkage varies in the same range of magnitude.

Short, practical guide on lightweight concrete (Kleine Leichtbetonkunde für die Praxis)

E. Vocke, Bauverlag GmbH., Wiesbaden, 1959, 143 pp.

Reviewed by H. H. Werner

This pocket sized book is a good resume of German practice and theory, on lightweight concretes. General characteristics, aggregates, mixes and mixing, placing, curing, and properties are treated; with particular attention to their application in the precast industry.

Graphical method of representing grading of aggregate (in Japanese)

Shunichi Mizuno and Tadashi Kasahara, *Journal*, Japan Society of Civil Engineers (Tokyo), V. 45, No. 10, Oct. 1960, pp. 13-18

Reviewed by Kiyoshi Okada

Describes a new method, and its application, of expressing the grading of aggregate graphically by a point in the particular polygon. By this method the fineness modulus and the specific surface of aggregate can also be expressed simultaneously with the grading.

We shall construct colored surfacings (in Czech)

E. Slachta, *Silnicni Doprava* (Prague), V. 8, No. 10, 1960, p. 8

Road Abstracts V. 28, No. 2, Feb. 1961

The effects of black ferrous pigments on the quality of concrete and mortars are studied. Tabulated tests results are given of investigations of the dynamic values and tensile strength in flexure after 28 days, of normal concrete and concrete containing 5 percent of black pigment. It is concluded that colored concrete for roads is durable, without glare, favors the thawing of snow and ice, and is not costly.

Mineral facts and problems—1960

Bulletin No. 585, Bureau of Mines, U.S. Department of the Interior, 1960, 1000 pp., $6 (Available from Superintendent of Documents, Government Printing Office, Washington 25, D.C.)

An encyclopedic reference on metals, nonmetals, and mineral fuels and the industries that produce them. Eighty-seven chapters cover the history, technology, and uses of minerals found in the United States. Of particular interest are those chapters on cement, lime and calcium, perlite, pumice, sand and gravel, stone, and vermiculite.

ASTM standards on mineral aggregates and concrete (with selected highway materials)

American Society for Testing Materials, Philadelphia, 9th Edition, 1960, 396 pp., $5.75

An attractive, hard-bound, compilation of ASTM specifications, methods of testing, and definitions related to mineral aggregates and concrete. Sponsors were ASTM Committee C-9 on Concrete and Concrete Aggregates and Committee D-4 on Road and Paving Materials. Subjects included are aggregates, concrete, concrete curing materials and expansion joint fillers, concrete reinforcing steel, and brick and block pavement materials. This collection should be useful to producers and consumers of these materials as well as to specification writers, testing and inspection personnel, and research and engineering organizations.

Rapid method for testing the resistance of cements to sulfate attack (Ein Schnellprüfverfahren für Zemente auf ihr Verhalten bei Sulfatangriff)

A. Koch and H. Steinegger, *Zement-Kalk-Gips* (Wiesbaden), V. 13, No. 7, July 1960, pp. 317-324

Author's Summary

A rapid method for testing the resistance of cements to attack by chemically aggressive waters is described. With the "small prism method," which can be applied to any kind of cement, portland cement and portland blast-furnace-cement can be tested with regard to this property within about 2½ months. Cements which, at the end of 56 days' exposure to sulfate, display no outward sign of damage may be classed as cements possessing an enhanced resistance to attack. A finer differentiation is provided by the sulfate absorption behavior and especially by the development of the relative flexural strength. The sensitivity of the rapid testing method and the agreement between the individual test results is illustrated with examples.

Pavements

Thermal stresses in concrete slabs for roads and runways (in German)

W. Klemp, *Strasse u. Autobahn* (Bielefeld), V. 11, No. 4, 1960, pp. 157-160

Road Abstracts V. 28, No. 2, Feb. 1961

Measurement of thermal stresses occurring in eight experimental concrete slabs of four different thicknesses at Hamburg airport are compared with previously calculated values.

Use of radioactive isotopes to test compaction of concrete road pavements (in Russian)

A. F. Ignatik, *Stroi. i dorozh. Mashinostro* (Moscow), V. 5, No. 2, pp. 8-26

Road Abstracts V. 27, No. 12, Dec. 1960

A new method for studying the compaction of freshly laid concrete has been successfully developed in the USSR. A radioactive isotope container and a cylinder containing a detector are let into the concrete at a set distance apart by a special vibrator device. The detector is connected to a radiometer which records electrically the intensity of radiation passing through the concrete. From the data obtained accurate estimates can be made of the degree of compaction at any depth with the minimum of delay. An accuracy of 5 percent is achieved which is virtually unaffected by the composition of the concrete. Tests can be completed in not more than 10 min.

Smooth surface from slip-form paving

R. M. Innis, *Civil Engineering*, V. 30, No. 11, Nov. 1960, pp. 44-47

Describes slip-form paving operations of the eight lane Harbor Freeway in California. Using a cement treated subgrade, an accumulative tolerance of 7 in. per mile was met.

Epoxy resin adhesive ties widening strip to old slab

Roads and Streets, V. 104, No. 2, Feb. 1961, pp. 65-67

Describes the widening of highways in Iowa with the use of epoxy resin adhesives to prevent cracks at the construction joint of the widening strip and old pavement by "welding" the concrete to the old.

Stresses in the corner region of thin bonded concrete road slabs

S. Koteswara Rao and V. Venkatasubramanian, *Journal of the Indian Roads Congress* (New Delhi), V. 25, No. 2, Dec. 1960, pp. 149-162

Highway Research Abstracts
V. 31, No. 4, Apr. 1961

Little information is available regarding load stresses in concrete road slabs constructed under Indian conditions, where thin slabs on a well consolidated water-bound macadam base are used. Experiments on load stresses in the corner region of a thin bonded concrete road slab constructed under field conditions were taken up at the Indian Institute of Technology, Kharagpur, and the results verified with existing theories. It was found that observed corner load stress agreed closely with Westergaard's theoretical values using his corner load formula. It was also found that in the vicinity of the corner bisector where the observed strains were maximum, the locus of the maximum strain and the shape of the actual load break were nearly straight lines, thus justifying Westergaard's assumption. The studies revealed that Westergaard's corner load formula can be used in the design of thin bonded concrete road slabs.

Bonded resurfacing and repairs of concrete pavement

William G. Westall, *Bulletin* No. 260, Highway Research Board, 1960, pp. 14-24

Discussion of bonded concrete construction and its various applications to pavements. Projects where this type of repair has been used are described. Essential steps in surface preparation and construction are illustrated. All the jobs discussed were constructed at airports.

Recent experience with and the knowledge gained from the construction of concrete roads in Germany (in German)

G. Streit, *Strassenbau* (Düsseldorf), V. 51, No. 5, 1960, pp. 277-279, 282-290

Road Abstracts
V. 28, No. 1, Jan. 1961

This illustrated account deals with subgrade and base, thickness of concrete surfacing, reinforcement, prestressed concrete roads, joints, mix proportioning, mixing and laying of concrete, and curing.

Diamond sawed joints in concrete pavement

Industrial Diamond Division, Engelhard Hanovia, Inc., 113 Astor St., Newark 2, N. J., 1961, 18 pp.

Presents the compiled data obtained in a nationwide field use investigation of concrete highway joint construction. This is probably the best accumulation and presentation of information on on present practice in sawing joints in concrete with diamond saws. The report compares sawed joints to other methods of joint formation and then discusses variables to consider before actual sawing operations. A section discusses the control of factors in the operation to be oriented for maximum sawing efficiency. This includes depth and width of joint equipment, selection of blade, and operating technique. A suggested specification is presented for sawed joints. Case histories are presented, briefly describing the type of work, aggregate, sawing operation, dimensions, and cost.

Fontenay-Trésigny experimental prestressed concrete pavement (La Chaussée éxperimentale en béton précontrainte de Fontenay-Trésigny)

R. Peltier, *Annales*, Institut Technique du Batiment et des Travaux Publics (Paris), Supplement No. 156, Dec. 1960, pp. 1330-1354

Description of the experimental prestressed concrete pavement on the Fontenay-Trésigny, France, bypass. Total length of the four sections was 1.86 miles. Various types of abutments, joints, and longitudinal and transverse prestressing methods were tested. Results of a few tests on effect of concrete temperature, sliding of the slabs on the subgrade, and the rheological behavior of the concrete are reported.

Prestressed concrete runway at Brussels airodrome (Piste en béton précontrainte de l'aerodrome de Bruxelles)

A. Paduart, *Annales, Institut Technique du* Bâtiment et de Travaux Publics (Paris), V. 13, No. 155, Nov. 1960, pp. 1205-1230

Reviews fundamental reasons for prestressing pavements and describes Belgian work on the experimental Zwartberg-Meeuwen Road and more particularly the longitudinal prestressing of the Brussels airfield runway using jacks and fixed abutments.

"Active joints" over channels in which the longitudinal prestressing cables were anchored and spaced at 330 m with temporary joints at 110 m. Initial prestress of the first bond (there were five) was 112 kg per sq cm, reduced after several minutes to 82 kg per sq cm. The 30 kg per sq cm difference was necessary to overcome friction. Transverse prestressing was by Freyssinet cables spaced at 1.75 m.

Continental concrete roads

D. R. Sharp, *Reinforced Concrete Review* (London), V. 4, No. 3, Sept., 1956, pp. 141-176
Reviewed by William R. Lorman

Deals with planning, design, and construction of postwar concrete highways in Belgium, Denmark, Germany, and Holland. The section on layout emphasizes the great use and success of detailed road planning. The next section points to the important influences of various factors (soil mechanics, steel reinforcement, joints, traffic loads, and concrete mixtures) on the design of base and slab; each of these factors is treated separately. The section on construction emphasizes the mechanized operations prevalent in the countries visited. Riding qualities, maintenance work, and experimental investigations are sketched briefly. The reader of this provocative paper will conclude that " . . . it pays to construct to a high standard of quality." Construction techniques comprise the 13 photographic illustrations; references consist of 11 items. The discussion covers a multitude of technical points raised by the 14 participants.

Precast Concrete

How screen block is shaping up

Richard S. Huhta, *Concrete Products*, V. 63: No. 10, Oct. 1960, pp. 24-31; No. 11, Nov. 1960, pp. 24-31

Illustrates the vast number of patterns that have been used for screen block.

Vertexload tables for large load pipes obtained by photoelastic investigation (Scheitellasttafeln für Grosslastrohre durch spannungsoptische Untersuchungen)

W. Gormann, *Beton und Stahlbetonbau* (Berlin), V. 55, No. 6, June 1960, pp. 136-139
Reviewed by Rudolph Szilard

The German concrete pipe industry has developed a new type of pipe Gross-Lastrohr capable of supporting three times the vertex load of a conventional pipe. To find out the actual stress distribution in the sections, photoelastic investigations have been used. The result of the investigation indicates that the critical points are at 20 deg from the vertical symmetry line. Based on these investigations a new more economical section has been developed.

Precast concrete bridge slabs to cut SP maintenance costs

Railway Age, V. 150, No. 6, Feb. 1961, pp. 18-19, 22

Reviewed by Aron L. Mirsky

Minimum maintenance cost was one of criteria determining selection of precast concrete bridge slabs and pile for six bridges on 9-mile line relocation in Texas.

Design, construction and stability of industrial structures built of precast reinforced concrete elements (Ueber die Gestaltung, Konstruction und Stabilitaetsverhaeltnisse von Mehrgeschossigen Industriebauten aus Stahlbetonfertigteilen)

Tihamer Koncz, *Die Bautechnik* (Berlin) V. 37, No. 1, 2, and 3, Jan., Feb., and Mar. 1960, pp. 29-36, 62-72, and 109-113

Reviewed by J. F. Leffmann

Reviews the general layout and typical modules, the shapes and types of precast frame elements, columns, girders, beams and floor sections, and the types of joints, and discusses their relative advantages in buildings of different sizes and under different live loadings, their fabrication, erection, and costs. Considers questions of analysis of multistory frames and stability problems, giving special attention to systems with inner joints. Eleven structures, designed by the author and others are use to illustrate the findings. A well written and well illustrated paper.

Reinforced concrete piling and piled structures

F. E. Wentworth-Sheilds, W. S. Gray, and H. W. Evans, 2nd Edition, Concrete Publications Ltd., London, 1960, 149 pp., $4

Concise information for designers, constructional engineers, and students concerned with the manufacture, driving, and design of precast concrete piles, the testing of soils, and the design of piled structures. Examples of methods of design according to British codes of practice and construction, and illustrations of many structures are given. Subjects which are additional or have been amplified include loading tests, the calculation of bearing capacities of piles, the design of sheet-pile retaining walls, and specification and quantities. The stresses occurring during driving are dealt with in detail. Methods of designing pile caps for single piles and groups of piles are given with numerical examples.

Prestressed Concrete

Addendum to the stress redistribution due to shrinkage and creep in case of multistrand prestressing (Zur Spannungsumlagerung durch Schwinden und Kriechen bei mehrstraengiger Vorspannung)

B. Vik, *Beton und Stahlbetonbau* (Berlin), V. 55, No. 8, Aug. 1960, pp. 185-187

Reviewed by Rudolph Szilard

Analytical methods for computing the stress redistribution produced by a shrinkage and creep are developed. To simplify the problem, the curve of the stress-creep factor is approximated by a parabola. The reliability of the simplified formulas is about 5 percent.

Various aspects of corrosion of reinforcement in reinforced and prestressed concrete (Les divers aspects de la corrosion des armatures dans les ouvrages en béton armé et précontraint)

M. D. Petrocokino, *Annales*, L'Institut Technique du Bâtiment et des Travaux Publics (Paris), V. 13, No. 153, Sept. 1960, pp. 916-936

A general discussion of the behavior of any metal immersed in an electrolytic solution is followed by the application of these theories to concrete in which large cells are formed by the reinforcement and the concrete. The different possibilities related to pH and properties of concrete as an electrolytic conductor are then brought out.

Finally the possible dangers which may be incurred in prestressing elements made of hard grades steel, such as anodic dissolution without rust and hydrogen embrittlement, are reviewed.

A table gives 55 factors influencing corrosion.

Recent developments in prestressed concrete in Belgium

D. VANDEPITTE, *Reinforced Concrete Review* (London), V. 4, No. 12, 1958, pp. 705-728
Reviewed by WILLIAM R. LORMAN

Presents principal features pertaining to design, model tests, and construction of four prestressed concrete structures: a viaduct, a pavilion, a taxiway, and a suspension bridge. Contains 21 figures (mostly diagrams) and two references.

Post-tensioned lightweight 3 in. warped shell spans 250 ft

Building Construction, V. 30, No. 9, Mar. 1961, pp. 14-19

This hyperbolic paraboloidical shell, supported on four corners, employed post-tensioning in ridges and edges to achieve minimum weight in the large warped shell. The shell was constructed of 95 lb per cu ft lightweight concrete averaging 3 in. thick with a compressive strength of 3750 psi.

In addition to using post-tensioning at the ridges and edges of the shell, foundation blocks at the corners were tied diagonally by two post-tensioned concrete cross tie members each 279 ft long.

Prestressed concrete resists fire damage

CHARLES C. ZOLLMAN, MARCELLO G. GARAVAGLIA, and AVIV RUBEN, *Civil Engineering*, V. 30, No. 12, Dec. 1960, pp. 36-41

Discusses the investigation of fire damage to a building framed of precast, prestressed concrete. After the above average, 5 hr duration of the fire many of the structure members were proved usable by simple and rapid means. The investigators checks loss of camber by comparing measured elevations with undamaged members or with the computed camber. Visual checks of crazing, cracking, spalling, and discoloration were then used and supplemented with Schmidt hammer tests. Of the members placed in the doubtful category, the two least likely to withstand test requirements were scheduled for load testing in the field.

Prefabricated hyperbolic paraboloid shells in prestressed concrete (in Dutch)

L. G. M. BREKELMANS and CH. J. DE VILDER, *Cement* (Amsterdam), Oct. 1960, pp. 974-975; Feb. 1961, pp. 87-89
Reviewed by JOHN W. T. VAN ERP

These shells are 61 x 8.5 ft and only 2 in. thick, with straight 7 mm prestressed wire reinforcement. They are the roof of a church and were transported by truck more than 60 miles. Fabrication under rigidly controlled conditions in the factory proved to be advantageous and also economical notwithstanding the difficulties of transportation.

The prestressed concrete road at Dietersheim (Spannbetonstrasse Dietersheim)

H. SCHECKE, *Strasse und Autobahn* (Bielefeld), No. 4, 1959, pp. 118-122
Reviewed by J. F. LEPPMANN

Describes design and construction of a 3000 ft long, 25 ft wide experimental road section which was completed in 1958. The 6.25 in. thick high strength (6400 psi) concrete pavement rests on 6 in. of bituminous gravel, topped with a bituminous sand leveling course. Bonding of concrete to the base was prevented by either a layer of talcum between paper or by two layers of Suprathen. The concrete was prestressed longitudinally in 500-ft sections with bundles of seven rods (0.4 in. diameter steel and 227,000 psi ultimate strength) at 30 in. centers, and prestressed transversely with single rods ($\frac{7}{16}$ in. diameter steel and 214,000 psi ultimate strength) at 16 in. centers. The 11 in. wide joints between concrete sections were filled with assemblies consisting of alternating layers of $\frac{5}{16}$ in. steel plates, standing upright, and profiled neoprene. Concrete deformations due to shrinkage, creep and temperature variations were observed at 21 points on each section. The check points served to evaluate the coefficient of friction of the pavement base. The costs of this type of pavement are discussed but not its performance under traffic.

Pretensioned concrete piles — Present knowledge summarized

T. Y. Lin and W. J. Tablot, Jr., *Civil Engineering*, V. 31, No. 5, May 1961, pp. 53-57

Summarizes the current status of knowledge of pretensioned concrete piles and points out areas that deserve further study. Discussion covers design, detailing, manufacture, driving, and applications.

Alternate design cuts cost of parking garage

Contractors and Engineers, V. 58, No. 3, Mar. 1961, pp. 52-55

A prestressed design cut costs over a conventional reinforced design on a parking garage in Beverly Hills, Calif. The main girders are precast T-sections, which nominally span 75 ft from the exterior columns to a central shear wall. The girders will be post-tensioned after the slabs are placed. Formwork is also discussed.

A guide to the B. S. code of practice for prestressed concrete

F. Walley and S. C. C. Bate, Concrete Publications, Ltd., London, 1961, 104 pp., $3

An important commentary on the British Standard Code of Practice CP 115 (1959), "The Structural Use of Prestressed Concrete in Buildings." The Code is printed in full and comments and explanations follow each clause. In particular those recommendations concerned with aspects of prestressed concrete construction which are not common to other forms of concrete construction are amplified. Headings are definitions and symbols, materials (especially the properties of prestressing steel), design stresses, the design of structural works subjected to pure bending or direct compression, fire resistance, methods of prestressing, practical considerations, inspection and testing. A concise bibliography is given and there are 12 tables in addition to those in the code, and 12 diagrams or charts containing valuable data for the designer particularly for eccentrically-loaded members.

Concrete piles designed for marine foundations

E. G. Paulet, *Civil Engineering*, V. 31, No. 3, Mar. 1961, pp. 51-53

Describes a different approach to concrete pile design used successfully for long piles on off-shore drilling rigs in Lake Maracaibo, Venezuela. The method permits longer piles, resulting in a saving of about 25 percent over long-column design. A representative pile with a 200-ton capacity is 100 ft long and has an average cross section of 24 in. The long column method would require an average cross section of 36 in.

Further developments of the Gifford-Udall prestressing system

R. E. D. Burrow and E. W. H. Gifford, *Reinforced Concrete Review* (London), V. 5, No. 4, Dec. 1959, pp. 221-246
Reviewed by William R. Lorman

The developments described in this paper supplement the techniques described previously by Gifford in the *Reinforced Concrete Review*, V. 3, No. 10, 1955, pp. 735-779 (refer to "Current Reviews," ACI Journal, *Proceedings* V. 55, No. 11, May 1959, p. 1233, for your reviewer's comments). The new techniques relate to large diameter strands and to the redesign of anchorages for multiwire cables. These improvements in prestressing were employed in the Narrows Bridge in western Australia. The section on the large strand system offers details concerning: prestressing strand, relaxation loss, load-deformation curve, anchor grip, tube unit, strand jack, control block, extension measurement, multiple-strand cables, friction problems, saddles, separators, anchorage zone, and experimental beam tests. A description of the Udall meter (based on a Wheatstone bridge arrangement of electrical resistance strain gages) precedes the second main section which is concerned with the tube anchorage system (for wire). Includes 12 diagrammatic figures and seven references; there is no discussion. This definitely is for the specialist in prestressing operations.

Prestressed concrete trusses save time and money for big plant

Engineering News-Record, V. 166, No. 5, Feb. 2, 1961, pp. 40-42

Describes the economic advantages in using prestressed concrete trusses for the redesign of a refrigeration plant in Secaucus, N.J. The trusses, up to 120 ft long, were precast cantilever sections supported on V-shaped columns. Top chords of the trusses were inverted T-sections, 27½ in. deep. The flanges were 24 in. wide and the bottom chords were 24 x 12 in.

Prestressed concrete (Part 1) (Spannbeton)

H. Herberg, B. G. Teubner Verlagsgesellschaft, Leipzig, 2nd Edition 1960, 342 pp.
Reviewed by H. H. Werner

This textbook on prestressed concrete covers the field thoroughly with up-to-date applications. Theories, design, materials, application, tests, and references are given fully, with many illustrations. All phases of basic prestressed concrete are well treated and a summary of many noteworthy applications is given, together with a comprehensive bibliography. This thorough first part on the subject promises to be supplemented by an interesting second part with practical applications.

Ultimate strength of prestressed concrete continuous beams (in Polish)

Stefan Kaufman and Jakus Mames, *Archiwum Inzynierii Ladowej* (Warsaw), V. 6, No. 3, 1960, pp. 397-45
Reviewed by S. Galezewski

The results of numerical examples of prestressed continuous beams in the ultimate states calculated in this paper have not confirmed the full validity of the plastic hinge hypothesis. In not one of the cases analyzed was the adaptation of the structure complete. At the same time, there was shown an effect of linear transformation on the ultimate load capacity of the structure. This effect can reach as much as 12 percent when large transformations are considered.

It was stated that other things being equal, the behavior of continuous prestressed beams in the ultimate state depends on the ratio of the elastic to the ultimate bending moment in a given section. Structures in which this ratio takes similar values for all critical sections reveals a small effect of adaptation, and in these cases the ultimate state is close to the elastic theory.

Properties of Concrete

Determination of the cement content of concrete by means of hydrometers (Bestimmung des Zement gehaltes im Beton mittels Aräometer)

Gentaro Tsuneyama, *Zement - Kalk - Gips* (Wiesbaden), V. 13, No. 8, Aug. 1960, pp. 354-360
Author's Summary

For determining the cement content of concrete, the measurement of the specific gravity was used; three hydrometers with ranges of measurement differing from, but close to, one another were employed for enhancing the accuracy. At the same time, the volume of the deposited sand was measured. Verification of the suitability of the method was carried out on concrete mixes with defferent mix proportions.

Effects of differential temperature on tall slender columns

D. A. Stephenson, *Concrete and Constructional Engineering* (London), V. 56, No. 5, May 1961, pp. 175-178
Author's Summary

When the surface of a concrete structure is subjected to strong solar radiation it absorbs heat, part of which passes into the surrounding air and part into the structure. The consequent rise of temperature causes strain near the surface of the concrete, the strain decreasing at increasing depths from the surface.

Recent trends in the design of bridges of several spans towards the use of continuous decks, supported on long

slender columns capable of absorbing movements of the deck by longitudinal flexure, give conditions in which the effects of differential temperature on the columns may result in considerable bending stresses. This article provides a guide to the assessment of such stresses.

Refractory concretes with barium-alumina cement as the bonding agent (in German)

AL. BRANISKI, *Revue Roumaine de Métallurgie* (Bucharest), V. 5, No. 1, 1960, pp. 165-185

BATTELLE TECHNICAL REVIEW V. 9, No. 10, Oct. 1960

The refractoriness and mechanical properties of barium alumina cement are proportional to the proximity of its composition to that of monobarium aluminate (SiO_2, Fe_2O_3, alkalis must be as low as possible). The conditions of solidification, refractoriness, resistance to sudden changes of temperature, and improvement of refractoriness were examined.

Symposium on effect of water-reducing admixtures and set-retarding admixtures on properties of concrete

Special Technical Publication No. 266, American Society for Testing Materials, Philadelphia, 1960, 246 pp., $7.50

The use of water-reducing admixtures and set retarders in concrete has grown continuously over the past 25 years so that ASTM and other agencies are drafting standard tests to govern requirements of these admixtures for concrete. This book contains the ten papers which were presented at a symposium sponsored by Committees C-1 and C-9 of ASTM. Four of the papers represent the joint contribution of four principal producers of admixtures. The remaining papers were prepared by representatives of consumer interests, research organizations, and the cement industry. A great amount of laboratory and field data are presented as well as descriptions of field experience with the use of various products in a wide range of applications. This book closes with a summary of the main points brought out in the symposium giving the effects of these admixtures and describing applications and procedures in use.

Effect of sand characteristics on the cracking of small bars of mortar in which shrinkage is restrained

F. A. BLAKEY and R. K. LEWIS, *Civil Engineering and Public Works Review* (London), V. 55, No. 644, Mar. 1960, pp. 389-393

This article reports on an investigation made to determine how cracking stress and strain of a mortar is affected by changes in the specific surface and maximum particle size of the aggregate.

The results show no important difference in cracking strain related to grading of the aggregate, and therefore support the hypothesis that the cracking starts in the cement paste rather than at the boundaries between aggregates and paste. Creep behavior appears to be independent of degree of restraint and properties of aggregates within the limits of the experiment.

When constant proportions of water, cement and sand are used the shrinkage has been shown to decrease as the voids in the sand increase.

The corrosion of steel in concrete (Zur Frage der Korrosion von Stahl im Beton)

G. REHM and H. MOLL, *Zement-Kalk-Gips* (Wiesbaden), V. 13, No. 5, May 1960, pp. 230-233

AUTHOR'S SUMMARY

In the short testing method used by Kaesche, Bäumen and Engell, inferences as to the danger of corrosion of steel in concrete are drawn from the trend of the current-voltage curves obtained with reinforced concrete electrodes. Owing to the complex influences that have a significant effect with regard to this method, it is not always possible to give an entirely satisfactory interpretation of the results of the measurements. Hence, it must be considered whether other test-

ing methods should not be adopted. Bars embedded in concrete and subjected to a low-voltage electric potential for fairly long periods, or test specimens exposed to different atmospheric conditions, would be suitable for the purpose. (Note: Rebuttals regarding this article follow in the same issue on pp. 233 to 239.)

Some effects of aging on the surface area of portland cement paste

C. M. HUNT, L. A. TOMES, and R. L. BLAINE, *Journal of Research*, National Bureau of Standards, V. 64A (Physics and Chemistry). No. 2, Mar.-Apr. 1960, pp. 163-169
Reviewed by ROBERT G. MATHEY

A hardened cement paste cured at room temperature had part of its evaporable water removed by vacuum drying and was studied. The surface area was shown to decrease with time depending on the amount of evaporable water left in the paste. This change is the opposite of that usually observed during hydration and probably represents some collodial growth phenomena analogous to aging observed in other collodial gels. Both water vapor and nitrogen adsorption measurements were used to show the effects of aging in cement paste.

Effect of alternate wetting and drying on the properties of portland blast-furnace cement concretes (in Japanese)

KAZUSUKE KOBAYASHI, *Transactions*, Japan Society of Civil Engineers (Tokyo), No. 72, Jan. 1961, pp. 15-22
Reviewed by KIYOSHI OKADA

Describes the results of experimental studies on the effects of alternate wetting and drying as well as continuous drying on the variations in volume, weight, and dynamic modulus of elasticity of portland blast-furnace cement concretes. Two types of portland blast-furnace cements of high (65 percent by weight) and low (45 percent by weight) slag content are used. Concludes that (1) as to drying shrinkage, no appreciable difference is found between normal portland and blast furnace-cement concretes, (2) when stored in water portland blast-furnace cement concretes show greater expansion than normal portland cement concretes, and (3) when subjected to alternate wetting and drying there are no remarkable differences between normal portland blast furnace cement concretes in the variations in volume, and weight, except in Young's modulus.

Efflorescence of cement mortar

RENICHI KONDO, TOSHIKAZU WAKUI, and TOSHIYOSHI YAMAUCHI, *Yogyo Kyokai Shi* (Tokyo), V. 68, No. 772, 1960, pp. 97-102
CERAMIC ABSTRACTS
V. 44, No. 4, Apr. 1, 1961

The effects of several factors concerning the composition of cements and mortars on the formation of efflorescence were studied. Its formation was decreased by reducing the alkali content of cement and by making the mortar more compact. The rates of water absorption, permeation, and drying are reduced by a lower water-cement ratio and a higher content of combined water. Efflorescence can be prevented by the use of coarse sand and by replacing part of the cement by powdered blast-furnace slag.

Recent research on the vibration of concrete

J. M. PLOWMAN, *Reinforced Concrete Review* (London), V. 4, No. 3, Sept. 1956, pp. 177-220
Reviewed by WILLIAM R. LOHMAN

A progress report which reviewer considers of great importance to concrete research personnel and of educatoinal interest to concrete construction engineers. Deals with physics of bond, mix design for vibrated concrete, and propagation and measurement of vibration. In the ensuing discussion, which reflects the views of 16 discoursers, Plowman recommends (among other features) a vibrational frequency of 6000 cycles per min and an acceleration of 5 g as the minima for ordinary in situ concrete from commercial sources. The discussion is three fourths the length of the paper proper and certainly contains much worthwhile information.

Evolution of creep and the properties of concrete (Evolution du fluage et des proprietes du béton)

M. MAMILLAN, *Annales*, Institut Technique du Bâtiment et des Travaux Publics (Paris), V. 13, No. 154, Oct. 1960, pp. 1017-1077

Report of a study of the results of tests on concrete test pieces subjected to permanent load. Factors evaluated included degree of curing time under load, age at time of loading, and dimensions of test piece.

Factors affecting bond between concrete and reinforcement

G. B. WELCH and B. F. J. PATTEN, *Constructional Review* (Sydney), V. 34, No. 2, Feb. 1961, pp. 31-39

AUTHOR'S SUMMARY

The relevant published information on bond developed between concrete and steel is reviewed and factors affecting bond discussed in some detail. Attention is drawn to the general lack of information relating concrete properties, other than compressive strength, to the bond strength. It is shown that important factors contributing to bond development are the settlement and sedimentation characteristics of plastic concrete and the volumetric changes of hardened concrete. The authors conclude that these and other concrete properties should be given more consideration when assessing bond strength.

Accelerated curing of concrete test cubes

THOMAS N. W. AKROYD, *Proceedings*, The Institution of Civil Engineers (London), V. 19, May 1961, pp. 1-22

Reports results of tests on accelerated curing of a large number of concrete test cubes to compare effects of accelerated curing by boiling and heating in an oven. A modified boiling method consisting of normal curing for 24 hr followed by 3½ hr of boiling was found to be simple and effective and well suited to laboratory operation schedules.

It was found that the 7 or 28-day compressive strength of cubes cured normally could be predicted by accelerating the curing and testing them 29 hr after casting. The effect of different brands of cement on the relationship between accelerated and normal specimens was found to be negligible whereas the aggregate had a much greater effect. At 28-day strengths over 4000 psi, errors in prediction increased.

Early age of concrete (Le premier age du béton)

L. VIRONNAUD, *Annales*, Institut Technique du Bâtiment et des Travaux Publics (Paris), V. 13, No. 154, Oct. 1960, pp. 1003-1016

Describes an apparatus intended to measure shrinkage of cement paste and concrete at early ages. Results show the effect of air movement at several velocities on shrinkage of paste and concrete specimens from 15 min up to 24 hr. Other factors studied were composition of the cement, water and cement content, gradation, and reinforcement.

Automatic freezing-and-thawing equipment for a small laboratory

WILLIAM A. CORDON, *Bulletin* 259, Highway Research Board, pp. 1-6

AUTHOR'S SUMMARY

Automatic freezing-and-thawing equipment in the laboratory provides a valuable tool in determining the performance and acceptance of proposed materials. Correlation between concrete strength and durability tests provides a means of establishing field control.

The cost and complicated nature of freezing-and-thawing equipment which automatically produces alternate cycles of freezing and thawing has discouraged extensive use of this test. Hand methods of placing specimens in freezing cabinets and thawing tanks are also costly and laborious. Hence, freezing-and-thawing tests are generally limited to research and performance tests in the larger laboratories.

Comparatively inexpensive automatic freezing-and-thawing equipment has been developed by the Engineering Experiment Station, Utah State University. This equipment produces up

to 9 cycles of freezing and thawing in 24 hr. The cost of the equipment may be within the budget of most small laboratories.

Paper discusses details of construction of the equipment and presents typical test results obtained with the development model of this equipment.

The technology and properties of heavy concrete (in Russian)

A. E. Desov, Editor, Gosstroiizdat (State Publishing Office for Literature on Building, Architecture and Building Materials), Moscow, 1959, 10r. 50k.

Concrete and Constructional Engineering (London) V. 56, No. 3, Mar. 1961

This book contains eight papers dealing with research on three aspects of heavy concrete, namely, compaction by vibration, shielding from radioactive sources, and autoclaving. The subjects of the papers are: efficient methods of vibrating concrete; methods of casting silicate mixtures by vibration; criteria for uniform intensities of vibrations with different frequencies; properties of heavy concrete designed for shielding from radioactive sources; design of special concretes of given density, strength, and workability; some questions on the technology and properties of autoclave concrete; factors affecting the strength of autoclaved concrete; and research on the modulus of elasticity of autoclaved concrete. The papers have been compiled after studying relevant American and European literature. Some of the conclusions are as follows:

Tables vibrating in two directions are recommended for casting large pieces. Limonite concrete has good resistance to frost. Reinforcement does not corrode in concrete made without the addition of calcium choride when irradiated with gamma rays, but in the presence of calcium choride corrosion is more intense than when the specimens are not irradiated. Bonding of reinforcement in concrete cured in an autoclave is generally no different from bonding in concrete cured by normal methods.

Steam treatment of concrete made with high-alumina cement at 174 C gives strengths at 1 day three times that normally obtained. Autoclaved concrete with compressive strength of over 7000 psi can be obtained from a mixture comprising 24 percent of high-strength portland cement, 16 percent of ground sand and 60 percent of unground sand, using a water-cement ratio of 0.45. The modulus of elasticity of autoclaved concrete is significantly lower than normally-cured concrete 1 to 3 days after steaming. Increasing the temperature of autoclaving lowers the modulus of elasticity by altering the composition of the hydration products and size of crystal by increasing the flaws and by drying the concrete more fully.

Structural Research

Reduced scale resistance models of caissons made of prestressed concrete of G2 and G3 reactors

N. Beaujoint, *RILEM Bulletin* (Paris), No. 10, New Series, Mar. 1961, pp. 25-34

Author's Summary

The calorific energy is extracted from the G2 and G3 reactors of the Marcoule Center by the circulation of carbonic gas at a pressure of 15 kg per sq cm. The enclosures, or caissons, subjected to this pressure and with a capacity of 3000 cu in. in which the reactors are situated, are made of prestressed concrete according to an original idea of the ACJB Bureau d'Etudes.

These works have been the subject of a study on reduced-scale resistance models carried out by the Service des Etudes et Recherches Hydrauliques d'Electricité de France, at the request of the Atomic Energy Commissariat. Paper describes the essential characteristics of the study: scale and number of models, their detailed description as well as those of the devices for measuring and organizing the tests. The results with regard to the idea of the study and the efficiency of the experiments are examined in conclusion.

Laboratory tests on the water impermeability and stability of block seams of concrete structures (in Russian)

T. G. Ginzburg, *Izvestiya Vsesoyuznogo* (Moscow), No. 56, 1956, pp. 144-155
Applied Mechanics Reviews
Oct. 1959

Five types of block seams were subjected to stability tests. The following deductions were made: (1) the incising of the surface of the concrete strongly diminishes the stability of the seam; (2) the same sort of incision but with subsequent rubbing in of a cement solution significantly increases the cohesion of concretes of equal strength; (3) vibrating the concrete increased its strength, compared with plugging, by 42 percent on average.

Tests on a one-tenth scale model of a hyperbolic paraboloid shell roof

L. L. Jones, *Magazine of Concrete Research* (London), V. 13, No. 37, Mar. 1961, pp. 21-36
Author's Summary

Tests which have been carried out on a one-tenth scale model of a hyperbolic paraboloid shell roof have given data from which the tie-bar force, corner reactions, deflection and stress distribution of the actual structure have been calculated.

In the interior of the shell, the stresses calculated are in good agreement with those calculated by the membrane theory. Near the edge beams both the forces and the moments increase rapidly; since, however, the edges of the shell were thickened, the maximum stresses in the structure under a working load of 60 lb per sq ft were only 250 psi compression and 100 psi tension. At this load the maximum deflection would be 0.5 in. The tie-bar force measured was only two-thirds of that predicted by the membrane theory, and equilibrium conditions revealed that the lower corners were carrying approximately 0.4 and the higher corners 0.1 of the total load.

The model shell had a load factor of at least 5.

Lateral pressure of fresh concrete on forms (in Dutch)

A. M. Witte, *Cement* (Amsterdam), Dec. 1960, pp. 1057-1065
Reviewed by John W. T. Van Erp

In an elaborate treatise the various aspects of the problem are reviewed: Test apparatus and results of tests in laboratories and on jobs are described, including the latest series of triaxial compression tests. Formulas are derived in which such items as specific gravity of concrete, speed of rise of concrete level in forms, angle of internal friction, and setting time of concrete are introduced. Good agreement between theoretical formulas and actual measurements is claimed.

Tests of composite beams with stud shear connectors

Charles Culver and Robert Coston, *Proceedings*, ASCE, V. 87, No. ST2, Feb. 1961, pp. 1-17
Author's Summary

Composite concrete-steel beams using L shaped stud shear connectors were tested. Both static and fatigue tests were conducted.

This series of tests has indicated that present specifications do not take into account the maximum useful strength on a static or fatigue basis of stud shear connectors. Further, it has been shown that loading these studs to ultimate does not alter the behavior of the complete section.

Optical tripod dilatometer, experimental results with cements and concretes (Dilatomètre a trépied optique, résultats expérimentaux sur ciments et bétons)

P. de Montera, *Annales*, Institut Technique du Bâtiment et des Travaux Publics (Paris), No. 145, Jan. 1960, pp. 47-58
Author's Summary

Present study gives the results of measurements of the coefficients of expansion of four cements obtained with the optical tripod dilatometer.

Brief description of the device is given in the second part, and then results of the measurements for three concretes are presented. These concretes differ only in the nature of their

aggregates, so that the influence of the latter on the expansion of the concrete is clearly brought out. The variations of expansion in relation to time and temperature are also determined.

An equation giving the expansion coefficient of the concrete as a function of the expansion coefficient of the cement and the aggregates is proposed.

Experimental analysis of structures, synthesis and criticism (Sintesis y critica del analisis experimental de estructuras)

JULIO RICALDONI, Publicaciones de la Facultad de Ciencias Fisicomatemáticas, Universidad Nacional de la Plata, Buenos Aires, V. 6, No. 4, Dec. 1959, 145 pp.

Reviewed by ROGER DIAZ DE COSSIO

A comprehensive summary of the most common experimental techniques for the experimental analysis of structures is given. This was compiled from a series of lectures given in Argentina by Professor Ricaldoni, Head, Institute of Statics, School of Engineering, Montevideo. Among the techniques discussed are theory of models, and photoelasticity. With regard to the applications for concrete, emphasis is placed on the problems introduced by cracking, scale effects, and nonelasticity.

Structural analysis of temperature stresses due to nonlinear temperature distribution in bars, slabs, and plates (Baustatische Berechnung von Temperaturspannungen Infolge Nicht Linearer Temperaturverteilung in Stäben, Scheiben und Platten)

GUNTHER RICKENSTORF, *Bauplanung-Bautechnik* (Berlin), V. 13, No. 11, Nov. 1959, pp. 498-503

Reviewed by J. F. LEPPMANN

Presents methods and examples for the analysis of stresses in unidirectional temperature fields.

In a prismatic body which is so oriented that a cross section is parallel to the field, stresses in this section can be computed as bending moments of a plate of the same shape under a loading which is represented by the curvature of the field at each point. In the special case of a parabolic heat diagram the representative plate load is uniform and conventional analytical methods can be applied.

An equation for temperature stresses in a plate oriented normal to the gradient of the temperature field is given by Timoshenko. The required integrations are facilitated by a simplified formula developed by the author, relating the heat propagation in a body to its thermic properties.

Bars are treated either as narrow prisms or as narrow plates. Nonstationary temperature conditions ("heat shock") are considered as successions of stationary ones.

General

Research: Engineering methods and materials

Bureau of Reclamation, U.S. Department of the Interior, 1960, 185 pp.

A report on research past, present, and future) conducted by the Bureau of Reclamation in engineering methods and materials for the purpose of improving engineering practices in water resource development. The term "research," as used in this report, covers studies and investigations of many types relating to the better uses of engineering materials and the improvement of engineering methods and processes. The report is divided into the following sections: organization for research; research personnel; facilities for research; research accomplishments; current research; and future research.

The largest part of the report is devoted to current research and a summary description is given of current projects. The general categories of research are divided into: concrete, cement and pozzolan, earth, hydraulic, structural, bituminous, special techniques, coatings and corrosion, rock foundations, riprap, evaporation reduction, plastics, sedimentation, weed control, water measurement, and lower cost canal lining.

Simplified engineering for architects and builders

HARRY PARKER, John Wiley & Sons, Inc., New York, 3rd Edition, 326 pp., $7

Up-dated version of an early edition first published in 1938. The usual materials of construction are covered with 66 pages being devoted to reinforced concrete. Design of concrete members is based on the ACI Building Code (ACI 318-56). Examples and solutions illustrate the methods used.

Industrial building, V. 1

Proceedings, Industrial Building Congress, Clapp and Poliak, Inc., New York, 1961, 232 pp., $10

Two papers are of interest to the concrete field, "The Use of Prefabricated Components," by William Schaem, and "Advances in the Use of Concrete," by J. D. Piper. Both are narrative accounts of the present status of recent developments in lift-slab, tilt-up, precast roof components, prestressed concrete, curtain walls, and concrete products.

Foundation failures

C. SZECHY, Concrete Publications, Ltd., London, 1961, 115 pp., $5

Reports the author's investigation of foundation failures in Hungary and those by others in other parts of the world. He divides causes of failures into four groups: (1) The absence of a proper investigation of the site or a wrong interpretation of the results of such an investigation. (2) Faulty design of the foundation or superstructure. (3) Defective temporary works or poor workmanship in the construction (4) Insufficient provision for exceptional natural phenomena such as thermal and biological conditions, rainfall, and floods greater than those hitherto recorded at the site. Most forms of ordinary and piled foundations are dealt with, and in particular the foundations of many types of buildings, bridges, industrial storage, marine and riverside structures are included.

Abstracts of theses—1953-1954

Massachusetts Institute of Technology, Office of Publications, 1960, 221 pp., $3.50

Contains abstracts of the 152 theses accepted for the Doctor's degree at Massachusetts Institute of Technology during the 1953-1954 academics year. Includes all departments of MIT.

Key to Smoley's

GEORGE F. WOLTERS, 1960, 75 pp., $1.50 (Available from George F. Wolters, P.O. Box 475, Ormond Beach, Fla.)

This booklet can be an aid to draftsmen, detailers, engineers, architects, and others using Smoley's parallel tables of logarithms and squares, and slopes and rises. The booklet containing graphic examples and solutions, and their applications to the tables has been simplified to such an extent that anyone with even a weak mathematical background can in short time become conversant with the tables for calculations in design and detailing.

Directory of university research bureaus and institutes

Gale Research Co., Detroit, 1960, 208 pp., $20

A reference guide to university research bureaus, institutes, centers, experiment stations, laboratories, etc., set up on a permanent basis and carrying on a continuing program of research in specialized fields.

The book contains 1500 listings in 250 fields including engineering, transportation, statistics, computer and data processing facilities, chemistry, geology, mining, education, public administration, Russian studies, explosives, community planning, and many others. Each listing includes the following data: (1) official name of bureau or institute, (2) address, (3) name of sponsoring college or university, (4) name and title of director, (5) year founded, (6) size of professional staff and nonprofessional staff, (7) title and frequency of issue of serial publications, (8) description of research program and activities, and (9) list of professional meetings, special programs and seminars.

Regulations of the German Board for reinforced concrete (Bestimmungen des Deutschen Ausschusses für Stahlbeton)

BERNHARD WEDLER (Editor), Wilhelm Ernst and Sohn, Berlin, 7th Edition, 1960, 544 pp., 19.80 DM

A compilation of more than 30 German regulations applying to reinforced concrete.

Reinforced concrete simply explained

OSCAR FABER, Oxford University Press, New York, 5th Edition, 1959, 88 pp., $2

This fifth edition is revised to follow the British Code of Practice CP:114 (1957) and by rearrangement of subject matter. A nontechnical treatise on the various aspects of reinforced and prestressed concrete design.

Masters of world architecture. Series I

George Braziller, Inc., New York, 1960, each book: 128 pp., each book: $4.95

Six monographs devoted to six modern architects. The topics range from aesthetics to details of structural design. It is particularly interesting to note that almost all of the architects discussed in the monographs have executed some outstanding works in concrete.

Pier Luigi Nervi

ADA LOUISE HUXTABLE

The text on Nevri is perhaps the most interesting for concrete engineers because of his extensive use of reinforced precast concrete, including fluted shells, lamella vaults, parabolic arches, fan shaped buttresses, ribbed and corrugated vaults. He has used these architectural forms in warehouses, airplane hangars, exhibition halls, factories, and stadiums.

Le Corbusier

FRANCOISE CHOAY

Most characteristic of Le Corbusier's work is his use of concrete in original ways. He uses it to support and enclose his buildings. He textures it, pierces and penetrates it, and uses it for both structural and architectural expression. Individual structures as well as town planning are described.

Frank Lloyd Wright

VINCENT SCULLY, JR.

The book on Wright gives a rough chronological comparison of some of his buildings and includes his early structures, use of concrete units in residential structures, and the dominant concrete curves of the Guggenheim Museum.

Ludwig Mies van der Rohe

ARTHUR DREXLER

Mies van der Rohe is discussed in terms of modern industrial construction with his emphasis on structural uniformity. Although emphasis is given to his steel and glass structures, several outstanding concrete projects are described.

Antonio Gaudi

GEORGE R. COLLINS

Barcelona's noted architect holds interest because of his use of hyperbolic paraboloidical surfaces. Some of his structures seem so incredibly daring that they look as if they might not stand, yet they were based on exhaustive scale model tests and were mathematically calculated with care. Many of the undulating vault surfaces he used are now used in thin shell concrete structures.

Alvar Aalto

FREDERICK GUTHEIM

Designer of several outstanding reinforced concrete structures, e.g., the tuberculosis sanatorium at Paimio, most of the structures of Alvar Aalto reflect his native Finland where brick is the predominant construction material because of the lack of steel for reinforced concrete.

OCTOBER 1961

NEWSLETTER

memo from the **President:**

Each Man on the Job Has a Part in Good Construction

Workmen have many things on their minds these days and probably there are few like the legendary mason who, when asked what he was doing, did not say he was shaping the stone, or building a wall, but said, and doubtless felt, he was "building a cathedral." It is worthwhile to foster the feeling in each workman that his part is important.

Modern construction is a cooperative operation involving many tasks and items. In view of its complexity, no detail can be slighted without affecting the final result. In view of the many other possibilities for imperfections, an individual may get the idea that care with what he is doing is unnecessary. Nothing is further from the truth.

The care and concern of every individual related to the job is essential to a quality of results in which all can take pride. In contributing his best workmanship to this end, even the routine worker can justly be proud of his part in the resulting well-completed structure.

The success of construction, and particularly of concrete construction, is up to each man from the engineer to the workman concerned with its many on-the-job and handcraft operations. That means you—and me—not somebody else!

Lewis H. Tuthill
President

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Kunze named chairman of ACI Committee 317

Walter E. Kunze, manager, Structural and Railways Bureau, Portland Cement Association, Chicago, has assumed chairmanship of ACI Committee 317, Reinforced Concrete Design Handbook. The Committee, discharged in 1959, has been reactivated to update the ACI special publication *Reinforced Concrete Design Handbook* in conformance with the revised ACI Building Code.

ACI technical committee appointments

Listed below are committee members who have recently accepted appointment to ACI technical committees. Included are *new* appointments only.

Committee 215, Fatigue of Concrete
Gordon G. Smith
Oklahoma State University
Stillwater, Okla.

Committee 716, High Pressure Steam Curing
Herbert E. Dux
Pennsylvania Glass Sand Corp.
Hancock, W. Va.

Committee 805, Application of Mortar by Pneumatic Pressure
H. L. Kalousek
Air Placement Equipment Co.
Kansas City, Mo.

Thomas D. MacLeod
Western Waterproofing Co.
Kansas City, Mo.

ACI worldwide membership brings about publication in several languages

In a move to make ACI publications of more value to members living outside the United States, the Institute is now including in its monthly JOURNAL, Spanish, French, and German translations of the explanatory synopsis which accompanies each technical paper. The full text of the papers will continue to be published in English only.

A large number of ACI members, about one in four, live outside the United States. The Institute's total membership of 10,434 is located in 90 countries.

ACI now offers two publications in Spanish, *Pinturas Para Hormigon* (Paints for Concrete) and *Proporcionamiento del Hormigon* (Recommended Practice for Selecting Proportions for Concrete — ACI 613-54). Plans are underway to issue Spanish editions of four more ACI publications this year.

Last year the 1958 edition of the *Concrete Primer* was published in Japanese and Italian. Publishers are the Japan Cement Engineering Association, Tokyo, Japan; and Editore Ulrico Hoepli, Milan, Italy. The Italian version is entitled *Tecnologia Del Calcestruzzo.*

During the past 50 years, Institute papers and publications have been translated and published in 20 languages other than English.

Birmingham

• • •

November 1-3

ACI 14th Regional Meeting

ACI's 14TH REGIONAL MEETING will be held at the Dinkler-Tutwiler Hotel in Birmingham, Ala., November 1-3.

Technical committee meetings are planned for Wednesday, November 1, with four technical sessions scheduled for November 2-3, followed by a field trip to the newly-constructed prestressed concrete structure carrying rail traffic over 19th Street in Birmingham.

The technical program (listed on p. 5) highlights latest advances in concrete technology and has been especially arranged with the interests of ACI members of the Alabama area in mind.

At the first general session on Thursday, November 2, speakers will discuss bridge and silo design and construction, as well as the design, fabrication, and testing of lightweight precast concrete framing. At concurrent materials and design sessions on Thursday afternoon, speakers will discuss blast furnace slag in concrete, chimneys, mass concrete, abnormal cracking, testing reinforced concrete floors, field testing, prestressed concrete design, and differential shrinkage effects in composite beams.

At the research session on November 3, papers will be presented dealing with beam tests, prestressed bridges, high strength reinforcing bars, and composite T-beams using epoxy resins as shear connector.

ACI President H. Tuthill, concrete engineer, California Department of Water Resources, Sacramento, will be the featured speaker at the Thursday luncheon. His subject will be, "Modern Resources for Better Concrete Work."

Clarence A. Barinowski, special consultant to Vulcan Materials Co., Birmingham, is general chairman for the 3-day meeting. Dovetailing "good old Southern hospitality" with a diversified technical program the planning committee has arranged social events and entertainment planned solely for the ladies, such as get-acquainted buncheon, a fashion luncheon, scenic tours, and shopping expeditions.

J. Shiers Jones, Connors Steel Division, H. K. Porter Co., Inc., is secretary of the local committee and Raymond E. Strickland, Strickland & Associates is secretary.

Committees and their chairmen, all from Birmingham area, are as follows:

Technical Program—J. H. Appleton, professor of civil engineering, University of Alabama; S. R. Millhouse, technical service manager, Universal Atlas Cement Co., and Wallace McRoy, consulting engineer, cochairmen.

Inspection Trips — J. H. Appleton, University of Alabama.

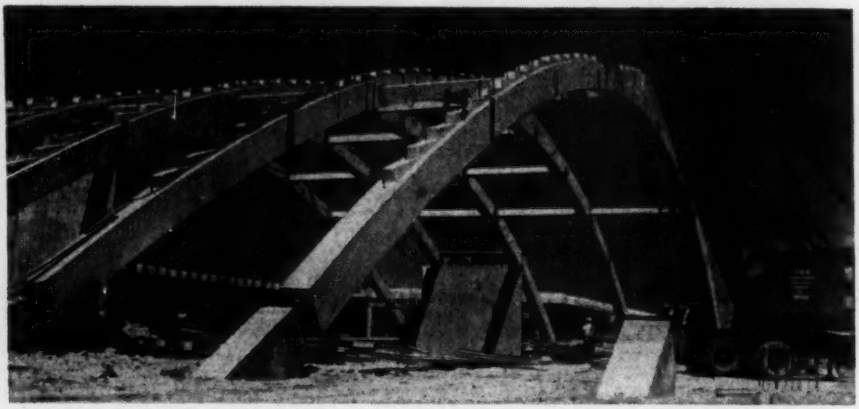
Publicity—Lloyd Belcher, Rust Engineering Co.

Registration—Frank E. Lindstrom, Sullivan, Long & Hagerty

Finance—Joseph H. Fox, J. H. Fox & Co.

Ladies' Entertainment—John Lanier, John H. Lanier Co.

Educational Contacts—J. H. Appleton, University of Alabama.



Each of the arches spans 160′ and is spaced 16′ center to center. The arch skeleton is braced by precast lateral members with welded connections. Gym floor space is 12,000 sq. ft., and there is a 10′ wide covered walkway on each side. Note "steps" and projecting dowels cast into arches for securing roof panels.

UNUSUAL PRECAST CONCRETE DESIGN FOR GYM ROOF

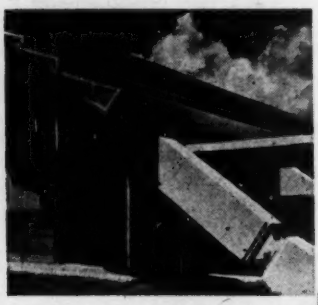
Modern precast concrete was imaginatively used to achieve an outstanding combination of function and beauty in Miami Central High's new gymnasium.

The three-hinged arches were cast on a concrete form at Precast Corporation's plant for minimum tolerances and maximum uniformity. In place, they rise directly from buttress footings supported on precast piling. Unusual precast roof slabs span the 16′ between arches. In addition to upward and downward legs for a shingle-type fit, the slabs also have a unique facing of glazed tile.

Lehigh Early Strength Cement was used for peak efficiency in the production of all precast units. In precasting the arches, for example, its use made it possible to turn out two arch-halves a week from a single form.

Lehigh Portland Cement Company, Allentown, Pa.

Closeup shows shape and ceramic tile finish of roof slabs. Each panel spans 16′, is 2′ thick, and 4′ wide including upward and downward legs. Special units at bottom act as rain gutter. Aqua colored high-glaze tile was applied by casting it integrally with the slabs as a form liner. Tiles were later grouted and given a coat of silicone waterproofing.



Miami Central High School Gymnasium

Architects:
Polevitzky, Johnson & Associates

Structural Engineers:
H. J. Ross Associates

Contractor: Thompson-Polizzi Construction Co.

Fabrication/Erection of Precast Concrete: Precast Corp.

All of Miami, Florida

View of completed gym speaks for its grace and practicality. Roof slabs were attached only by welds to dowels in the arches, solving problems involving thermal expansion and contraction. Then joints were covered with sheet copper strips set in a caulking compound.

LEHIGH CEMENTS

Membership Promotion—James Faircloth, University of Alabama, assisted by Fred M. Hudson, Auburn University; and Clive J. Luke, Wilmore, Hudson & Luke.

TECHNICAL PROGRAM

THURSDAY MORNING, NOV. 2

First General Session

Silo Design and Construction

Lloyd R. Belcher and George Shalas, Rust Engineering Co., Birmingham

Design, Fabrication, and Testing of Lightweight Precast Concrete Framing

Ross H. Bryan, consulting engineer, Nashville

Design and Construction of the 19th Street Underpass

G. A. Heft, consulting engineer, New Orleans

CONCURRENT TECHNICAL SESSIONS

THURSDAY AFTERNOON, NOV. 2

Materials Session

Development of Blast Furnace Slag Used in Concrete

Earl W. Fowler, Birmingham Slag Division, Vulcan Materials Co., Birmingham

Concrete Chimneys

Max Zar, Sargent & Lundy, Chicago

Optimum Air Content for Interior Mass Concrete

William O. Tynes, U.S. Army Engineer Waterways Experiment Station, Jackson, Miss.

Investigation of Abnormal Cracking in Highway Structures in Georgia and Alabama

Calvin C. Oleson, Portland Cement Association, Skokie, Ill.

Design Session

Load Test Requirements for Reinforced Concrete Floor Construction

D. E. Parsons, Building Research Division, National Board of Standards, Washington, D.C.

The Use of Field Testing as a Design Aid

R. C. Elstner, The Engineers Collaborative, Chicago

Service Load Design Versus Ultimate Load Design for Prestressed Concrete

Narbey Khachaturian, University of Illinois, Urbana

Differential Shrinkage Effects in Composite Beams

D. E. Branson, University of Alabama, University, Ala.

FRIDAY AFTERNOON, NOV. 3

Research Session

The Behavior of Reinforced Concrete Beams with Closely Spaced Reinforcement

J. R. Romualdi and G. B. Batson, Carnegie Institute of Technology, Pittsburgh

Development Length of High Strength Reinforcing Bars

Phil M. Ferguson and J. Neils Thompson, University of Texas, Austin

Experimental Investigation of Load Distribution Characteristics of a Prestressed Concrete Multibeam Bridge

C. R. Bramer, F. L. Morasdith, and E. P. Loppacker, North Carolina State College, Raleigh, N.C.

The Action of Composite T-Beams Using Epoxy Resins as the Shear Connector

J. D. Kriegh, University of Arizona, Tucson

FRIDAY AFTERNOON, NOV. 3

Field Trip

19th Street Underpass

GENERAL SESSIONS ARE OPEN TO ACI MEMBERS AND THEIR FRIENDS ENGAGED IN THE CONCRETE FIELD.

CONCRETE PERFORMANCE REPORT:

Controlled performance concrete was needed for TWA Terminal roof shell

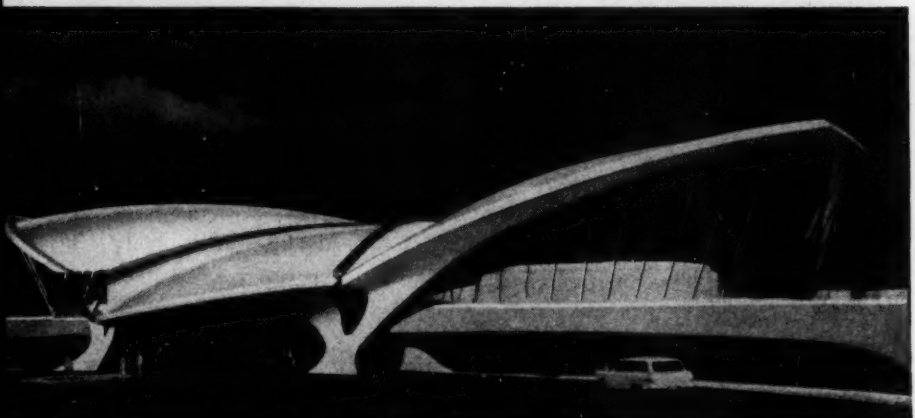
The 5500 ton roof of Eero Saarinen's distinctive TWA terminal at Idlewild Airport is an arch cantilever design—with four monolithic concrete shells, joined integrally at the centerplate, flowing out of four sculptured concrete buttresses.

In proportioning the 4000 psi conventional concrete for the buttresses and lightweight concrete for the shells, special consideration was given to these factors: (1) lowest possible unit water content and low cement content to minimize shrinkage (2) careful control of slump which had to be varied for different areas of the shell (3) precise control of setting time to assure a completely monolithic structure for each shell and to permit scheduled removal of counterforms (top forms) to allow finishing the surface to the final architectural contours. Also, in the buttresses and thick sections of the shell, a low rate of heat evolution was required to minimize thermal stresses.

Preliminary work showed that these objectives were best met with POZZOLITH concrete. The trap rock concrete for the buttresses contained 606 lbs. cement and POZZOLITH Retarder. The Norlite lightweight concrete for the shells contained 614 lbs. of cement, POZZOLITH Retarder and 23 gallons of water for the 5" slump mix required under the counterforms. By simply regulating the amount of

TWA

POZZOLITH Retarder for each individual load of concrete, the set was retarded, as required, in the range of one to four hours. The two batching stations got their instructions by field telephone from the concreting superintendent on the roof.



EERO SAARINEN'S MODEL of new TWA unit terminal, Idlewild Airport, New York. Two-and-one-half year project scheduled for completion in late 1961. 50,000 sq. ft. lightweight concrete roof shell—supported by only four concrete buttresses—weighs 5500 tons, contains 3200 cu. yd. of POZZOLITH concrete and 500 tons of steel. Architect: Eero Saarinen & Associates • Supervising Resident Architect: Ralph Yeakel • Consulting Engineers: Ammann & Whitney—Boyd G. Anderson, Senior Partner in Charge • General Contractor: Grove, Shepherd, Wilson & Kruge, Inc.—Kenneth P. Morris, Field Project Manager • Testing: Haller Testing Laboratories, Inc.

NEW $15 MILLION SCULPTURE IN CONCRETE is as functional as it is breathtaking. Thickness of the lightweight concrete roof shell ranges from 8" at the edges to 44" at the centerplate. Each of the four huge shells is a continuous monolithic section.

TERMINAL

Concrete was placed in a round-the-clock operation averaging 27 hours per shell. Removal of counterforms proceeded on schedule. The concrete finished easily, with good texture and no plastic cracking. There were no shrinkage cracks in the entire shell. 28 day strength tests averaged 5000 psi.

Master Builders fieldmen worked closely with the resident architect, engineer, contractor and ready-mix producer at the job site to achieve the common goal of maximum quality at lowest cost through controlled concrete performance.

Write for your free copy of the detailed "TWA Terminal Concrete Performance Report".

The Master Builders Company
Division of American-Marietta Company
Cleveland 18, Ohio
World-wide manufacturing and service facilities

Our 50th Year

MASTER BUILDERS. POZZOLITH*

P-1048

*POZZOLITH *is a registered trademark of The Master Builders Co. for its ingredient for concrete which provides maximum water reduction, controls rate of hardening and increases durability.*

FOUR CONCRETE BUTTRESSES support the massive shell. There are no internal columns. Buttress section is shown below after form stripping. Buttress surfaces will later be bush-hammered to the texture selected by Architect Eero Saarinen.

Asst.—Kenneth F. Morris, Field Project Manager • Testing: Maher Testing Laboratories, Inc.

Large Span Gymnasium Roof

The men's gymnasium at Indiana State Teachers' College in Terre Haute is attracting special interest because of its long roof span. With a prestressed folded plate concrete roof extending 165 ft (155 ft between columns), it is believed to be the longest single span structure of this kind yet built.

The column-free design is of particular advantage for a gymnasium, allowing flexible space for various sports and an unobstructed view for spectators. An area 210 x 155 ft, which will normally be used as three gyms for physical education classes, can be converted into one large room for intercollegiate basketball games and similar events. Telescopic remote-controlled 30-row bleachers will pull out over two of the gym floors to provide seating for 5000 people.

An adjoining structure will provide space for three classrooms, 14 offices, a 72 x 49 ft gymnastic's floor, a 49 ft square wrestling and boxing area, and a 75 ft long Olympic size swimming pool.

In addition to functional advantages, the combination of folded plate design and prestressing is proving economically attractive. Cost of the roof installed is approximately $6.36 per sq ft.

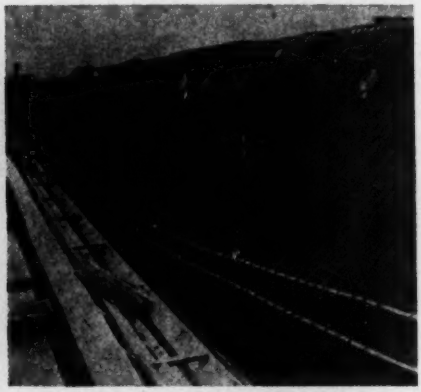


Photo Courtesy Portland Cement Association

Reinforcement being tied for folded plate over gymnasium

Folded plate design

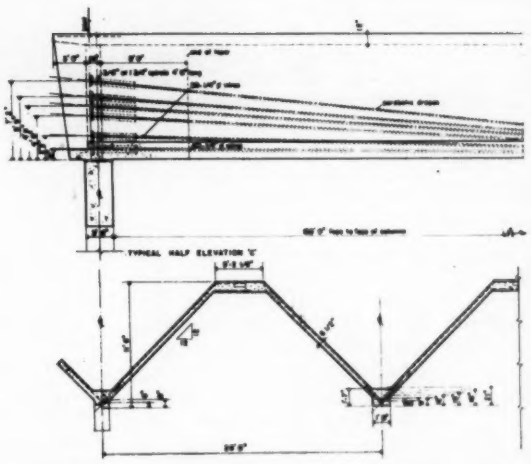
The folded plate design consists of eight configurations 26 ft across and 11 ft 6 in. high, extending crosswise of the building. To get maximum re-use of forms, a 12 in. construction joint was located in each top flange, and the closure strip cast after the adjoining sections were placed and tensioned. The top and bottom flanges are 12 in. thick, while the inclined sections are 6½ in. thick.

A stiff mix of concrete (2½ in. maximum slump) was required to provide high quality concrete and to facilitate placement on the 45 deg slope of the folds. In addition to metal tendons for prestressing, conventional reinforcement was used. The web sections were placed without the use of a top form.

Post-tensioning was used, and a strength of 4000 psi required of the concrete within 4 days after placement. Except for the two outside bays, each inclined web was equipped with six tendons, one containing 40 wires and the others 28 wires each. The wires, of high tensile steel, were ¼ in. in diameter. The Swiss BBRV post-tensioning system was used, and because of the length of the span, alternate tendons were tensioned from opposite ends to equalize the pressure. Each wire received an initial stress of 168,000 psi.

The stressing caused an upward deflection of 0.9 in. at the span center

Typical roof details of folded plates for men's gymnasium at Indiana State Teachers College



line and an inward movement of the bottom flange of 0.4 in. at each end. Neoprene pads between folded plate and girder accommodated this movement.

The gymnasium was designed by the architectural firm of Miller, Vrydagh & Miller of Terre Haute. Homer Howe, structural engineer; general contractor, J. L. Simmons Co., Indianapolis.

ADVANCE REGISTRATION

To: Mr. Frank E. Lindstrom, Registration Chairman
14th ACI Regional Meeting
c/o Sullivan, Long & Hagerty
801 Fifth Ave., North
Birmingham, Alabama

1. I plan to attend ACI's 14th Regional Meeting at the Dinkler-Tutwiler Hotel, Birmingham, Ala., Nov. 1-3. Please preregister me. ☐

2. My wife will accompany me. ☐

☐ **ACI member** ☐ **Nonmember**

I understand I am to make my own room reservations with Mr. Ira M. Patton, Dinkler-Tutwiler Hotel, 20th Street at 5th Ave., Birmingham 3, Ala.

Name .....

Address .....

City .....

CHAPTERS

Hawaii Chapter joins ACI ranks with formal charter presentation

ACI President H. Tuthill, concrete engineer, Division of Design and Construction, California State Department of Water Resources, Sacramento, made the official presentation of the charter to the Hawaii Chapter on June 7 at a dinner held in Waikiki.

The new officers and directors of the Hawaii Chapter were introduced and plans for future work of the chapter were discussed.

Alfred A. Yee, president, Alfred A. Yee and Associates, structural engineers, Honolulu, is chapter president with Tsutomu Izumi, superintendent of building, City and County of Honolulu, vice-president; and J. Bruce Vesey, president, Construction Products, Ltd., Honolulu, secretary-treasurer.

Directors for the chapter include: Edwin L. Bauer, architect; Tetsuo Harano, principal highway design engineer, State of Hawaii; Fujio Matsuda, chairman, Department of Civil Engineering, University of Hawaii; George H. McPherson, manager, Prestressed Concrete, Inc.; Nathaniel Potter, Jr., sales manager, Hawaiian Cement Corp., and S. Don Shimazuro, structural engineer. All of the directors are from the Honolulu area.

While President Tuthill was in Hawaii he addressed an invited group of engineers on the subject "Some considerations in getting good concrete work." The talk, sponsored by the ACI Hawaii Chapter, was held at the University of Hawaii and was attended by engineers from the construction branches of the Corps of Engineers, Navy, Air Force, State and County, and private engineers.

Reinforced concrete expert to address Michigan Chapter at first fall meeting

Paul F. Rice, technical director of the Concrete Reinforcing Steel Institute, Chicago, will present an expert's view of the new developments in the use of reinforcing steel for concrete construction at the first fall meeting of the ACI Michigan Chapter, Wednesday, October 11,

Mr. Rice, former technical director of ACI, continues to be active in a

ACI President Tuthill presents the charter for the ACI Hawaii Chapter to Alfred A. Yee, newly-elected chapter president at a dinner meeting at Waikiki

number of ACI committees. He is well known in Michigan engineering circles, having served 5 years as structural engineer for the Michigan District, Portland Cement Association.

Membership in the Michigan chapter is open to structural engineers, contractors, concrete products manufacturers, and other persons working in the concrete industry who live in Michigan or Lucas County, Ohio.

Southern California Chapter holds first fall meeting

ACI Southern California Chapter opened the 1961 fall season on September 21 at the Rodger Young Auditorium in Los Angeles with Kenneth D. Cummins, ACI technical director, as guest speaker.

Mr. Cummins spoke on formwork for concrete structures basing his discussion on the report on "Formwork for Concrete" prepared by ACI Committee 622 and published in the March, 1961, ACI JOURNAL.

Mr. Cummins discussed various phases of the committee report, explaining the need for a standard for formwork in the interest of both economy and safety. He showed slides illustrating collapses or failures which have occurred during construction, attributable to inadequate formwork for concrete. Slides were also shown illustrating the formwork features of a number of interesting concrete structures, including terminal, hangar, bank, and church buildings.

Hawke opens consulting office in San Francisco

James P. Hawke, internationally known consulting structural engineer, has opened a consulting engineering office in San Francisco. Mr. Hawke has just completed supervision of the design of the $188 million Volta River Hydroelectric Project in Ghana, Africa, now under construction.

Locally Mr. Hawke is known for his work in directing the engineering design of large industrial projects such as the Lockheed Navy Polaris Missile Plant at Sunnyvale, Calif., and for hydroelectric and irrigation works such as the Tri-Dam Project of the Oakdale-South San Joaquin Irrigation District.

His planning and design assignments in the international field have included the $100 million Karnafuli Dam in East Pakistan now nearing completion as well as hydroelectric and other major heavy construction projects in Brazil, Turkey, Afghanistan, Italy, Canada, Peru, India, and Kuwait.

THANKS TO PAST PRESIDENT AND MRS. FRANK KEREKES, a sturdy pin oak now graces the landscaped lawn just off the southeast corner of the ACI headquarters building in Detroit. It replaces a giant oak that was removed when Seven Mile Road was widened in front of the Institute building.

The "Kerekes Oak" was a most appropriate gift from the Dean. During the past 23 years he has been a "sturdy oak" in the structure of ACI activities. He has generously participated in executive and technical committee work, contributing his knowledge, time, and wholehearted cooperation.

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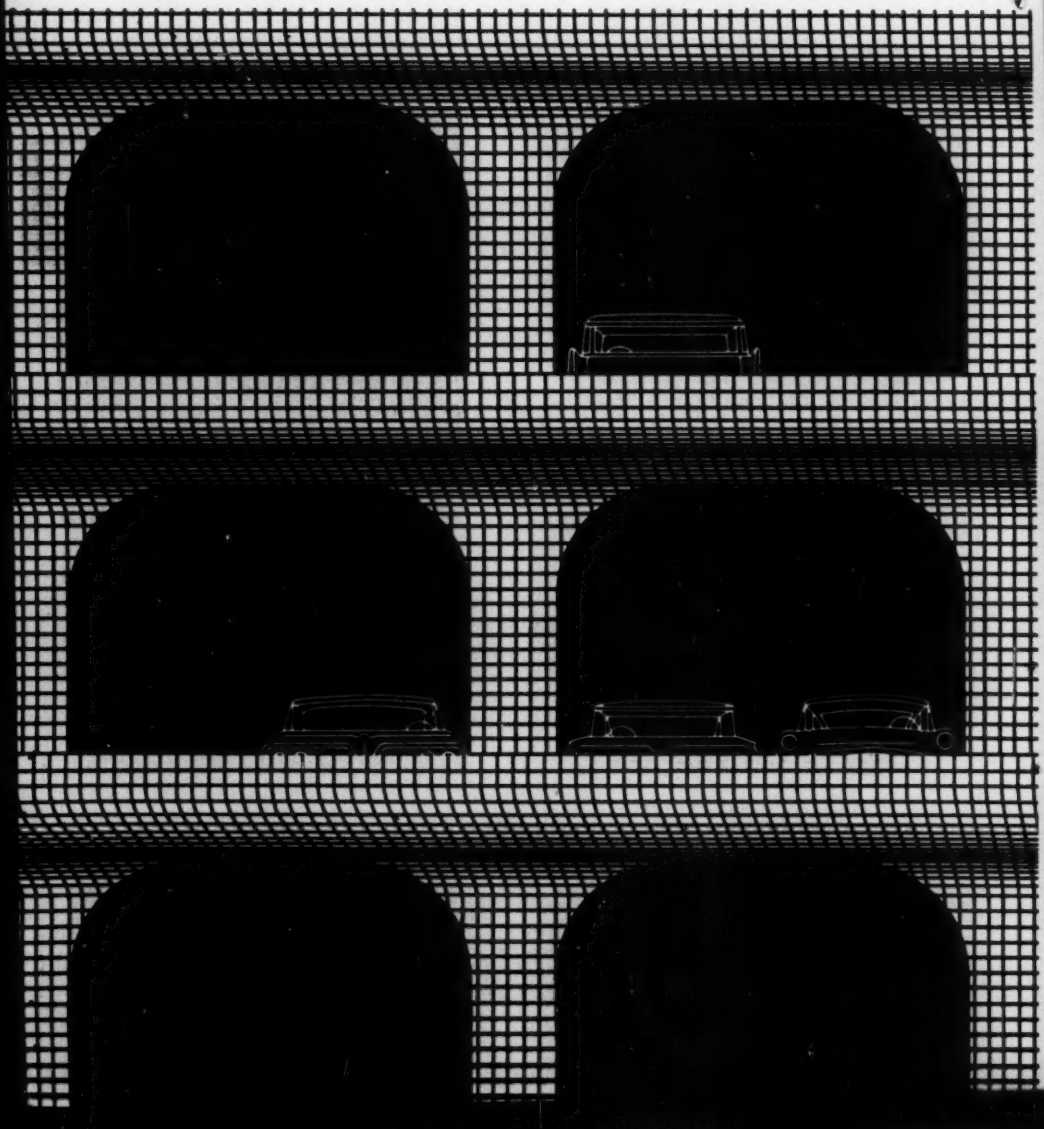

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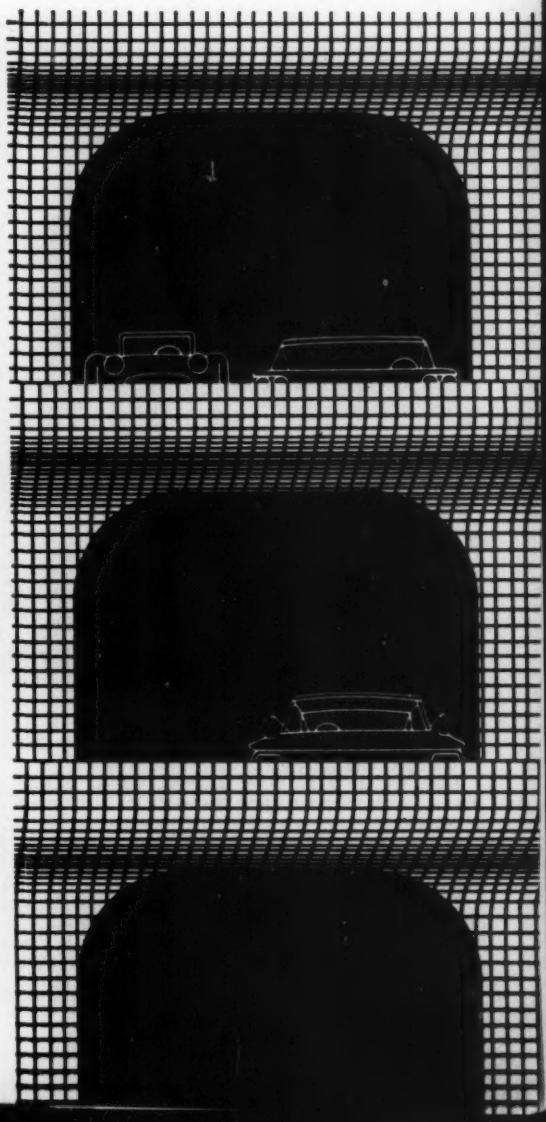

PCA fellows attend Purdue highway engineering program

As an aid to America's need for new and better highways, 19 instructors and professors of highway engineering from throughout the United States and Canada were enrolled in the Graduate School of Purdue University last summer for an 8-week special program of study in highway planning and pavement design. Each man received a free-grant fellowship sponsored by the Portland Cement Association, Chicago.

The students, many of whom hold advanced degrees, were able to select from 5 courses of study dealing with

The PCA Fellows who met at Purdue University this summer and several of the staff are shown in the above photograph.

Front row (l. to r.): W. A. Vaughan, Virginia Military Institute; R. J. Hensen, Fort Lewis College; Michael Larson, University of Minnesota; J. J. Schuster, Villanova University; W. J. Fogarty, University of Miami.

Second row (l. to r.): J. F. McLaughlin, Purdue; R. K. Palmer, Rensselaer Polytechnic Institute; W. A. Cordon, Utah State University; M. L. Andersen, South Dakota State College; G. R. Thomas, West Virginia University; W. G. Heslop, University of British Columbia.

Third row(l. to r.): H. L. Michael, Purdue; C. B. McCarthy, Tulane University; V. T. Christiansen, Utah State University; E. C. Sword, Lehigh University; J. J. Breen, University of Connecticut.

Fourth row (l. to r.): C. F. Scholer, Purdue; E. L. Fowler, University of Alberta, Canada; John Myers, The Citadel; J. A. Bullington, Tennessee Polytechnic Institute; W. C. Vodrazka, Mississippi State University.

Last row (l. to r.): W. L. Dolch, Purdue; W. G. Mullen, University of Maryland; E. G. Worth, Purdue.

various aspects of planning, design, and materials depending on their interests. Courses in geometric design, highway planning and economics, highway and airport pavement design, cement, aggregates and concrete, and in the basic physical and chemical properties of materials were offered.

Selection of the 19 Fellows was made from a list of over 60 applicants on the basis of their academic record, demonstrated professional ability, and the relationship of the program to the applicant's area of activity.

Six ACI members are among the PCA Fellows and teaching staff pictured on p. 14: J. F. McLaughlin, professor of civil engineering, Purdue University, Lafayette, Ind.; W. A. Cordon, professor, Department of Civil Engineering and Irrigation, Utah State University, Logan; E. C. Sword, professor of civil engineering, Lehigh University, Bethlehem; C. F. Scholer, instructor, Purdue University; E. L. Fowler, associate professor of civil engineering, University of Alberta, Canada; and W. G. Mullen, professor of civil engineering, University of Maryland, College Park.

Industrial Building Congress held in New York Coliseum

The second Industrial Building Exposition and Congress was held at the New York Coliseum, September 25-28. The twin events provided a common forum where problems of new construction and modernization were discussed, bringing together top management personnel from virtually every classification of industry. Plant engineers, architects, and consultants formed a large part of the visiting group.

Nineteen sessions of the Congress covered almost every aspect of new building and modernization. Exhibits of some 70 companies covered two floors of the Coliseum showing recent developments in structural materials and components, mechanical products, various types of equipment, and miscellaneous services.

King elected CEC president

Harold P. King, partner in the consulting firm of King, Benioff and Associates, Sherman Oaks, Calif., has been elected president of the Consulting Engineers Council. He succeeds Hueston M. Smith, St. Louis, Mo.

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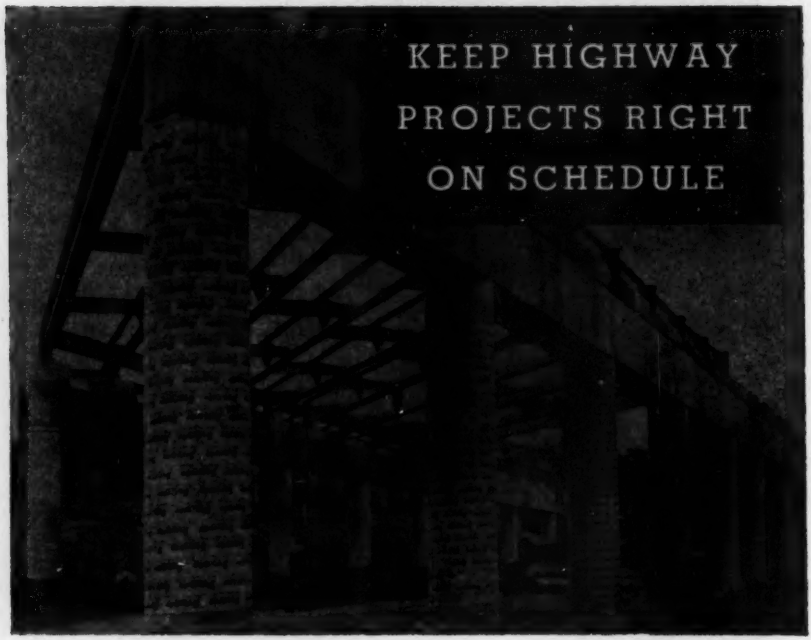

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Bates named director of NYU's University Valley

A. Allan Bates, formerly vice-president for research and development at the Portland Cement Association, Chicago, has been named the director of University Valley, New York University's new 1000-acre research and educational center in Sterling Forest, N. Y.

Plans to create University Valley, which will be surrounded by a number of industrial research laboratories, were announced last December by NYU and the Sterling Forest Corp. In addition to the construction of education and laboratory facilities, arrangements are being made to build homes for University faculty and staff members. Now being developed as a research and residential community, Sterling Forest consists of 30 square miles of property near Tuxedo, N. Y.

Dr. Bates joined PCA in 1946. From 1938 to 1946 Dr. Bates was manager of chemical, metallurgical, and ceramic research at the Westinghouse Electric Corp., Pittsburgh. During the previous 10 years he was a professor of metallurgical engineering at the Case Institute of Technology, Cleveland.

Dr. Bates has had broad international experience in industrial research. He has carried out a number of assignments for industry and government in Latin America and Europe. Last year he was chairman of an American delegation that made an extended tour of the Soviet Union.

In addition to arts and sciences degrees from Ohio Wesleyan University and Case Institute of Technology, he holds a doctorate in science from the University of Nancy, France. He also has received honorary doctorates from Rose Polytechnic Institute and Stevens Institute of Technology.

Long active in ACI executive and technical committee work, Dr. Bates is currently serving on the ACI Board of Direction. Last February during the ACI annual meeting, he was presented the Kennedy Award "for untiring efforts as chairman and member of the ACI Building Committee." He has just completed a term as national president of the American Society for Testing Materials and continues as a director of ASTM. He is a member of the executive committee of the Building Research Advisory Board of the National Academy of Sciences and serves on several advisory committees to the National Bureau of Standards. Dr. Bates is also active in a number of other professional societies.

Highlights from cement and concrete sessions featured during ASTM annual meeting

At the session on cement, held during the 64th annual meeting of the American Society for Testing Materials, Atlantic City, N.J., June 25-30, the "Significance of Selected ASTM C-1 Tests," was discussed by experts in their respective fields. Four papers were presented describing the evolution of the ASTM method of test for false set of portland cement; comparison of the mortar method and the paste method; method of test for calcium sulfate in hydrated portland cement mortar; and method of test for potential sulfate resistance of portland cement.

Other speakers at this session discussed the investigation of concrete materials for a major project in Western Canada; mechanism of grinding aids; and an improved automatically controlled adiabatic calorimeter for determining the temperature rise of mass concrete.

Committee C-1 on Cement

Several revisions and proposed new specifications for apparatus and methods were among the subjects discussed at the meeting of Committee C-1 on Cement. Several revisions were acted on for changes in the specification for Portland Cement (C-150), and for Masonry Cement (C-91), which will

bring corresponding Federal and ASTM specifications more nearly in line.

Two concrete sessions

New methods of test for evaluating concrete; new methods for repair; studies on the correlation of different methods of test; studies on the evaluation of fly ash, its economy and its uses in concrete; and modern concepts of control were highlighted at two sessions on concrete.

An extensive discussion of experimental short-time loading tests made on portland cement concrete cylinders to study relative strain-sensing capabilities of mechanical and resistant strain gages was presented.

Universal Atlas appoints Murphy to managerial post

Richard J. Murphy has been appointed technical service manager at the New York office of the Universal Atlas Cement Division, U.S. Steel Corp. He will service the New York, Philadelphia, Boston, and Albany sales territories in a technical capacity.

Mr. Murphy joined Universal Atlas as a technical service engineer in the Philadeplhia office in 1953. In 1958 he was transferred to the line sales staff, covering central and eastern Pennsylvania, a position he has held until his present managerial appointment.

LOOKING AHEAD

Oct. 4-5, 1961—Annual Meeting, National Slag Association, Pocono Manor Inn, Pocono Manor, Pa.

Oct. 7-10, 1961—Western Building Industries Exposition, Great Western Exhibit Center, Los Angeles

Oct. 16-20, 1961 — Annual Convention, American Society of Civil Engineers, Hotel Statler, New York

Oct. 15-19, 1961—7th Annual Meeting, Prestressed Concrete Institute, Brown Palace and Cosmopolitan Hotels, Denver, Colo.

Oct. 16-17, 1961—Annual Meeting, The Carolinas Ready Mixed Concrete Association, Inc., Jack Tar Poinsett Hotel, Greenville, S. C.

Nov. 1-3, 1961 — 14th Regional Meeting, American Concrete Institute, Dinkler-Tutwiler Hotel, Birmingham, Ala.

Nov. 14-17, 1961—16th Short Course for Industry Technicians, National Sand and Gravel Association and National Ready Mixed Concrete Association, University of Maryland, College Park, Md.

Nov. 28-30, 1961—Fall Conferences, Building Research Institute, Mayflower Hotel, Washington, D.C.

Dec. 5, 1961—First Quality Concrete Conference, University of South Carolina, Columbia, S. C.

Dec. 5-7, 1961—Fall Conferences, Building Research Institute, Shoreham Hotel, Washington, D.C.

Jan. 15-18, 1962—42nd Annual Convention, National Concrete Masonry Association, Americana Hotel, Bal Harbour, Fla.

Feb. 5-9, 1962—32nd Annual Convention and Siennial Show, National Sand and Gravel Association and National Ready Mixed Concrete Association, Conrad Hilton Hotel, Chicago

Feb. 11-15, 1962—Annual Meeting and Exhibition, National Crushed Stone Association, Conrad Hilton Hotel, Chicago

Mar. 12-15, 1962—58th Annual Convention, American Concrete Institute, Brown-Palace Hotel, Denver, Colo.

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ASCE extends invitation to ACI members to attend 1961 annual convention

The annual convention of the American Society of Civil Engineers will be held October 16-20 at the Hotel Statler Hilton, New York City, with the technical program largely devoted to the theme, "Metropolis—1980."

Gardner M. Reynolds, chairman, ASCE annual convention, has issued a special invitation to ACI members to attend any or all of the technical sessions.

Highlights of the technical program include 18 papers on construction, structural engineering, and engineering mechanics to be presented at the Construction Session. A research session on structural engineering is also scheduled.

Other major sessions will be devoted to city planning, sanitation, water, transportation, and power development.

CSI to sponsor institute at U of W, October 19-20

The University of Wisconsin Extension Division in cooperation with Region 7 of the Construction Specification Institute will hold an institute on "Preparing Construction Contracts and Specifications" on October 19-20, on the Madison campus of the University of Wisconsin.

The institute will be a 2-day educational meeting in which current information on preparing contracts and specifications will be presented. Guest lecturers will be architects and engineers from all parts of the country, men tops in their fields. Topics to be covered include: contract documents, legal principles as applied to construction, bidding procedures, bonding of bidders, and insurance for the construction industry.

The committee planning the program and making all arrangements consists of: Robert G. Burkhardt, R. G. Burkhardt and Associates, Chicago, chairman of the educational committee, Region 7, CSI; Harold L. Olsen, Olsen and Evans, Madison, president of the CSI Madison Chapter; and Edward O. Busby, University of Wisconsin Extension Division, Madison. Inquiries concerning the institute registration procedures should be directed to Mr. Busby.

Sherod joins Alpha staff

Joseph O. Sherod has joined Alpha Portland Cement Co., Easton, Pa., as director of manufacturing. In his new capacity, he will supervise the operation of all Alpha's eight plants.

Before coming to Alpha, he had served as general manager of production, Lime Division at Warner Co., Philadelphia. Previously he had been associated with Permutit Co., U. S. Army Ordnance Dept., Socony Mobil Oil Co., and DuPont Co.

EXHIBIT SPACE

58th Annual Convention

Denver

Choice exhibit space is now available for ACI's 58th annual convention at the Brown Palace Hotel, Denver, March 12-15.

The planning committee expects a registration of about 800 persons. Registrants will include structural and project engineers, architects, contractors, and public officials responsible for the construction of concrete buildings, highways, bridges, dams, precast and prestressed concrete products.

For complete information and space reservations, suppliers to the industry are requested to write Orley O. Phillips, exhibits chairman, 831 Fourteenth St., Denver 2, Colo.

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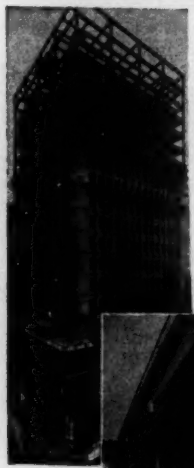

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Connolly joins Boston engineering firm

William H. Connolly has joined Cleverdon, Varney and Pike, consulting engineers, Boston, as director of the Prestress Design Division of their structural engineering department. He was formerly chief structural engineer for Goldberg, LeMessurier and Associates, and is author of the recently published book, *Design of Prestressed Concrete Beams.*

Comann appointed PCA Iowa District engineer

David H. Comann has been appointed district engineer of the Portland Cement Association's Iowa District. Mr. Comann, who will be headquartered in Des Moines, succeeds the late Fred F. Loy.

Mr. Comann joined PCA in 1953 as Housing and Products Engineer in its Iowa office. In 1956 he was assigned as general field engineer in Northeastern Iowa, a position he held until late 1959 when he was named office engineer for the District. Prior to joining PCA, he was head of the civil design section of the Stanley Engineering Co. in Muscatine, Iowa.

Chain Belt assigns Powell to key post

The appointment of William T. Powell as manager of its construction machinery section's western division has been announced by Chain Belt Co., Milwaukee. He succeeds S. Y. Warner who has been named western regional manager of the firm's industrial section.

In his new position, Mr. Powell will be responsible for all west coast construction machinery activities for Chain Belt including equipment sales and administration of the firm's Los Angeles plant. He will headquarter at the Los Angeles plant.

Mr. Powell, who joined Chain Belt in July of this year, is well known in west coast industrial and oil field trade.

White joins Cornell staff

Richard N. White has been appointed assistant professor of civil engineering at Cornell University and will assume teaching and research duties in structures this fall.

Dr. White recently obtained his PhD degree in civil engineering from the University of Wisconsin. During the past few years he has been engaged in teaching and graduate study at the University of Wisconsin, and also in consulting work in Madison.

Powers awarded honorary degree by U of T

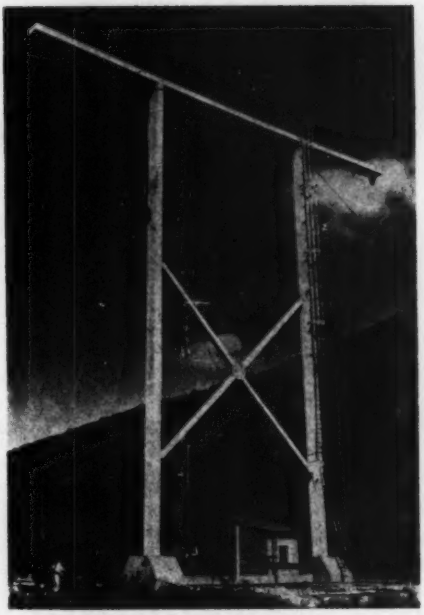
T. C. Powers, research counselor, Portland Cement Association, Chicago, has received an honorary Doctor of Science degree from the University of Toledo for "unique contributions to research in civil engineering" and "unusual service to humanity."

In honoring the veteran research scientist, the Board of Directors of the Ohio school particularly commended his furtherance of colloidal-physical theories and experiments, and his advancement of concrete and silicate science.

Since joining the Portland Cement Association in 1930 as an assistant engineer in its Research Laboratories, Mr. Powers has earned national recognition as the author of more than 50 technical papers dealing with various aspects of cement and concrete technology. He was awarded Honorary Membership in ACI at the 1961 convention in St. Louis.

In 1932, 1938, and again in 1947, he was awarded the ACI Wason Medal for exceptional research papers. In 1957 he was recipient of the ASTM Sanford E. Thompson award.

Mr. Powers served for 14 years as manager of PCA's Basic Research Section, prior to being named research counselor in 1958. He is currently a member of ACI Committee 212, Admixtures.



PRESTRESSED TRANSMISSION TOWER —

This 74 ton prestressed concrete transmission tower was erected as part of Project EHV (extra high voltage) being conducted by the General Electric Co. near Pittsfield, Mass.

Tower is 99 ft high and constructed of precast prestressed hollow box sections. The cross beam is 93 ft long. Sectional dimensions are 3 ft x 1 ft 6 in. for the legs and ⅛ in. smaller in the short directon for the beam. Cross bracing members are 12 x 12-in. solid sections, also precast and prestressed, joined at the intersection and legs with steel inserts. The precast sections were shipped 135 miles to the site by truck.

The legs are set 4½ ft into sockets in the tower bases. Each cast-in-place base rests on four piles.

The tower was supplied to the project, which is testing a variety of materials and tower designs, by the Prestressed Concrete Institute. Designers were Schupack and Zollman, Stamford, Conn. Manufacturer was C. W. Blakeslee and Sons, New Haven, Conn.

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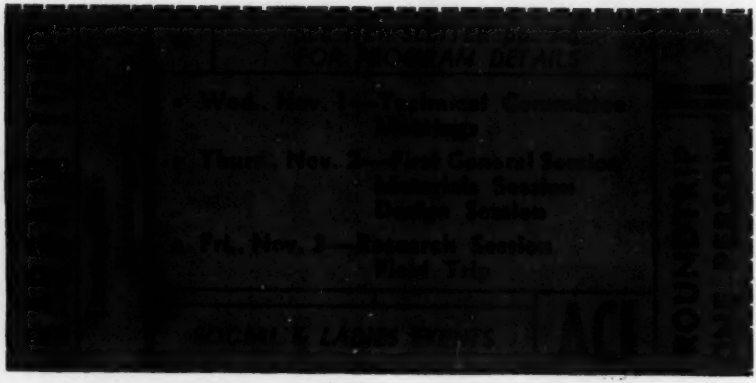
14th Regional ACI Meeting

November 1-3, 1961



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New York Trap Rock enters lightweight aggregate field

New York Trap Rock Corp., West Nyack, long a crushed stone producer, has entered the lightweight aggregate field. The firm expects to be serving the New York metropolitan area with a high grade structural lightweight aggregate by the spring of 1962.

A new $2 million plant will be constructed on Rondout Creek in the town of Ulster near Kingston, N. Y., where a subsidiary company will produce Nytralite at an anticipated production rate of 500,000 cu yd per year.

Universal Atlas opens new sales ofice

Universal Atlas Cement Division of U.S. Steel Corp., New York, has opened a new sales office at Port Everglades, Fla.

Appointement of a four-man sales staff has been announced: Hardy Magrath, formerly executive secretary of the Concrete Research Institute of South Florida, Inc., manager-South Florida sales and office manager; James L. Baker, salesman, with headquarters at Miami; Gilbert L. Hataway, salesman, with headquarters at West Palm Beach; and George W. Kelly, salesman, with headquarters at Port Everglades.

Clifton L. Schandelmayer has been named superintendent in charge of the cement distributing facilities at Port Everglades.

Karl appointed Nytralite v-p

William W. Karl has been appointed vice-president of Nytralite Aggregate, Inc., a newly formed subsidiary of New York Trap Rock Corporation. He is former president of Lehigh Materials Co., Tamaqua, Pa.

Mr. Karl is currently headquartered at the parent organization's main office in West Nyack, assisting Paul M. Hedley, Nytralite president. The two officials are making preliminary plans for the erection of a new $2 million plant on Rondout Creek in the town of Ulster near Kingston, N.Y.

Mr. Karl joined Lehigh Materials in 1955 as sales manager, developing the use of lightweight aggregate in structural concrete. The firm became a subsidiary of the Warner Co. in 1957 and in April 1959, Mr. Karl was named president of Lehigh Materials.

He is currently a member of ACI Committee 213, Properties of Lightweight Aggregates and Lightweight Aggregate Concrete.

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By S. CHAMPION, Consulting Engineer. Covers many types of failure, diagnosis, testing, inspection. Stresses repair, waterproofing, flooring problems. 1961. In Press

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Alpha appoints Ellis plant manager

William E. Ellis has been appointed manager of the Birmingham plant of Alpha Portland Cement Co., Easton, Pa. He will direct all operations at the Birmingham plant where he has served as plant engineer since 1956.

Universal Atlas Cement acquires Ponce Products

The Universal Atlas Cement Co., division of United States Steel, New York, announces the purchase of Ponce Products, Inc., including Ponce's storage and handling facilities at Port Everglades and the SS Florida State, a self-unloading bulk cement carrier, for approximately $4.5 million.

Universal spokesman said that the firm will buy its cement from Ponce Cement Corp., which has a plant in Puerto Rico. The cement will be produced in compliance with Universal specifications and will be sold under the "Atlas" brand name.

Harper and Jackson elected Permanente vice-presidents

Claude E. Harper, vice-president and general manager, Kaiser Gypsum Co., Inc., and William P. Jackson, vice-president and general manager, Glacier Sand & Gravel Co., have been elected vice-presidents of Permanente Cement Co., Oakland, Calif.

The announcement was made by Permanente President Wallace A. Marsh, following a recent meeting of the company's board of directors. At the same meeting Kenneth J. Samuels was elected assistant secretary of Permanente Cement, Permananente Steamship Corp., Glacier Sand & Gravel, and Permanente Trucking Co.

Kaiser Gypsum and the Seattle-headquartered Glacier Sand & Gravel are wholly-owned subsidiaries of Permanente. Both Mr. Harper and Mr. Jackson are veterans of the Henry J. Kaiser companies.

Kimber retires from CCI

George Harold Kimber retired from the Calcium Chloride Institute on August 1, following 14 years as its president, and the past year as board chairman. He will continue to serve in an advisory capacity on special projects when necessary.

As former head of the executive committee for the Calcium Chloride Association, Mr. Kimber became managing director of the CCA in 1946, and served in that capacity until 1951 when, under his direction, the CCA was reorganized and incorporated as the Calcium Chloride Institute. From 1951 to 1960 he served as first president of CCI.

Hutchens named C-W manufacturing manager

The Carter-Waters Corp. of Kansas City, Mo., has appointed Harold R. Hutchens to the post of general manager of the company's extensive concrete products manufacturing facilities in Kansas City.

Mr. Hutchens left a private engineering practice in 1957 to join Carter-Waters as a research engineer specializing in precast and prestressed concrete. He is registered in the states of Missouri, Kansas, and Iowa and is currently serving on the board of directors of the Prestressed Concrete Institute and the Expanded Shale, Clay, and Slate Institute. In addition to his ACI membership, Mr. Hutchens is a member of ASCE, the American Railway Engineering Association, and the Kansas City Engineers Club.

Errata

The following correction should be made in the discussion of "Rheological Behavior of Hardened Cement Paste under Low Stresses," which appeared in the September 1961 JOURNAL.

p. 1799—7th line from the bottom change "K" to "$\overline{K}$."

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Fig. 1—Concrete compression test specimens are usually cylindrical with a length equal to twice the diameter. The standard size is 6 in. in diameter and 12 in. long if the coarse aggregate does not exceed 2 in. in nominal size. The molds shown are of metal and have a machined metal base plate. For easy specimen removal, the molds are split longitudinally and held in the closed position by quick acting clamps. **Fig 2**—The specimens are formed by placing the concrete in the mold in three layers of approximately equal volume. Each layer is uniformly rodded with 25 strokes of the tamping rod. **Fig. 3**—Where voids are left by the tamping rod, the sides of the mold are tapped to fill the voids. After the top layer has been rodded, the surface of the concrete is struck off with a tamping rod or straight edge. **Fig. 4**—Specimens are covered with a glass or metal plate to prevent evaporation. They are removed from the mold at the end of 24 hr and stored under moist conditions. Photos courtesy Soiltest, Inc.

Puls retires from USBR

Louis G. Puls, chief designing engineer, U.S. Bureau of Reclamation, Denver, Colo., has retired after more than 30 years of Government service.

As chief designing engineer since 1958, Mr. Puls has directed the design of dams, power and pumping plants, canals, and other structures for the Bureau's water resource development projects in the 17 western states and Alaska.

Notable designs carried out under his direction include the 520 ft high concrete arch Yellowtail Dam, now under construction on the Missouri River Basin Project in Montana, and the Trinity, Clear Creek, and Spring Creek powerplants being built on the Central Valley Project in California. He directed the design of the Twin Buttes Dam, a large earthfill structure under construction on the San Angelo Project in Texas, and the designs for the storage and power features to be constructed on the Curecanti Unit of the Colorado River Storage Project in Colorado.

Prior to his present position, Mr. Puls was assistant chief designing engineer, serving in that capacity since 1956. He was head of the Concrete Dams Section from 1952-1956, supervising the designs of many large concrete dams including Glen Canyon and Flaming Gorge dams, and major features of the Colorado River Storage Project now in advance stages of construction.

Mr. Puls plans on entering private practice as a consulting engineer.

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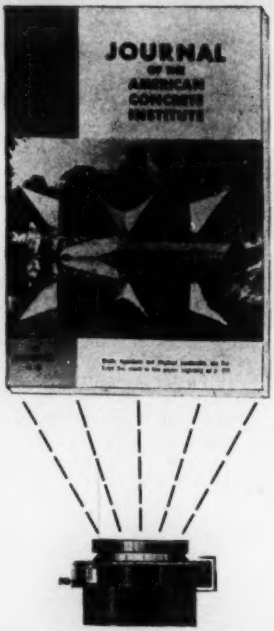

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H. Mogens Krabbe

H. Mogens Krabbe, works manager of the Bellingham, Wash., plant of Permanente Cement Co., Oakland, Calif., recently died at the age of 58.

Following graduation in 1924 from the University of Washington, Mr. Krabbe held engineering positions involved in the construction or operation of cement plants throughout the eastern states. He returned to Bellingham in 1932 as assistant superintendent and plant engineer of the Olympic Portland Cement Co., Ltd. He became works manager on the retirement of his father, Adolph Krabbe, in 1948, and remained in that position following the purchase of Olympic by Permanente in 1958.

Ayers appointed to sales staff of Dewey and Almy

Charles M. Ayers has joined the Dewey and Almy Chemical Division, W. R. Grace & Co., Cambridge, as sales engineer in the Construction Chemicals Department. He will work out of the firm's Chicago office.

Permanente Cement to build centers in Hawaiian Islands

Modern bulk cement distribution plants are being established in Hilo, Hawaii, and Kahului, Maui, by Permanente Cement Co., Oakland, Calif.

The twin plants will each have a storage capacity of 2000 bbl of cement and will be the first such installations on the neighbor islands.

The two facilities, representing a combined investment of more than $200,000, are expected to be in operation by December. Bulk cement from Permanente's new Waianae producing plant on Oahu will be barged to the Hilo and Kahului facilities.

The Hilo and Kahului facilities will raise the number of Permanente distribution plants to 13. The distribution plants are located throughout Hawaii, Guam, Alaska, California, Oregon, and Washington.

Honor Roll

Point System

1 point for Student; 2 points for Junior; 3 points for Individual; 4 points for Corporation; and 5 points for Contributing.

Jan. 1—Aug. 31, 1961

In the past 8 months over 300 members have added their names to the ACI Honor Roll by proposing new members for the Institute. Have you signed up a new member recently? Among your colleagues working in concrete, there are probably several excellent ACI prospects. Why not tell them about ACI today. Let's keep the membership trend spiraling!

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The Board of Direction approved applications in the following categories: 57 Individual, 1 Corporation, 11 Junior, and 26 Student, making a total of 95 new members. Considering losses due to deaths, resignations, and nonpayment of dues, the total membership now stands at 10,434.

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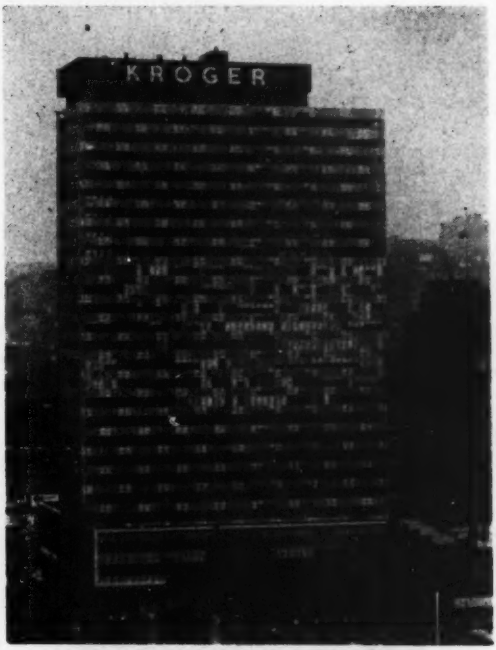
RAIL STEEL
REAL STRENGTH



CONTRACTOR—George A. Fuller Co., Dallas, Texas
ARCHITECT—Hedrick & Stanley, Ft. Worth, Texas

THE new 27 story Kroger Building, Cincinnati, Ohio, is constructed of concrete with 1532 tons of rail steel reinforcing bars. Originally this building was designed with structural steel frame but later changed to a modified "Ultimate Strength Design" of concrete construction and Rail Steel Reinforcing Bars for lower cost.

The American Building, shown at right of Kroger Building, was built in 1927 with structural frame and Rail Steel Reinforcing Bars.

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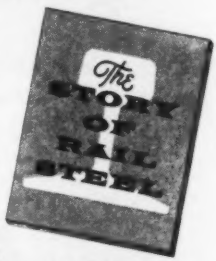

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RAIL STEEL BAR Association

38 South Dearborn St., Chicago 3, Illinois

50th Anniversary of the Rail Steel Bar Association, 1911-1961

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Tools, Materials, Services

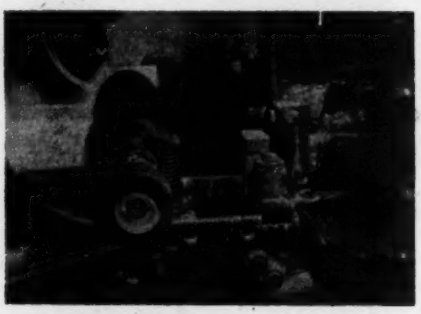
Under this heading note is made of producer literature and products of presumed technical interest to ACI users of tools, equipment, materials, accessories, and special services.

Besser acquires Clayton generator franchise

The Clayton Mfg. Co., El Monte, Calif., has franchised the Besser Co., Alpena, Mich., manufacturers of concrete block machinery, for world wide distribution of Clayton Steam Generators to the concrete block and allied industries. The Besser Co. will provide Clayton generators for new block manufacturing installations as well as for replacement of obsolete steam equipment in existing plants. —Besser Co., Alpena, Mich.

Pallet grinder

A coated abrasive sleeve, mounted across the frame of the block machine and rotated at high speed, cleans pallets as they pass in their normal cycle, ending build-up and pro-



ducing uniformly-dimensioned blocks according to the developer.

Photo shows device installed on a typical block-making machine. Pallets pass under the revolving abrasive belt and are cleaned immediately before each use. Spring loading of the pillow blocks permits the roll to adjust automatically should two pallets accidentally be fed together. A 5-hp motor drives the grinder through V-belts to give the abrasive a surface speed of approximately 5500 ft per min.

Tested in many plants and on thousands of pallets, manufacturer claims the abrasive sleeve is both long-lived and aggressive in its grinding action, keeping the cost of cleaning a pallet to a small fraction of a penny.—Bergen Machine & Tool Co., Inc., Franklin Ave., Nutley, N. J.

Portable wet-or-dry gun

The WD Gun, Airplaco's latest entry into the concrete gunning field, is a fully portable gun capable of both wet or dry gunning and pressure grouting. Designed as an all-around gun, it is said to have outstanding flexibility in that it can change over instantly from one method to the other.

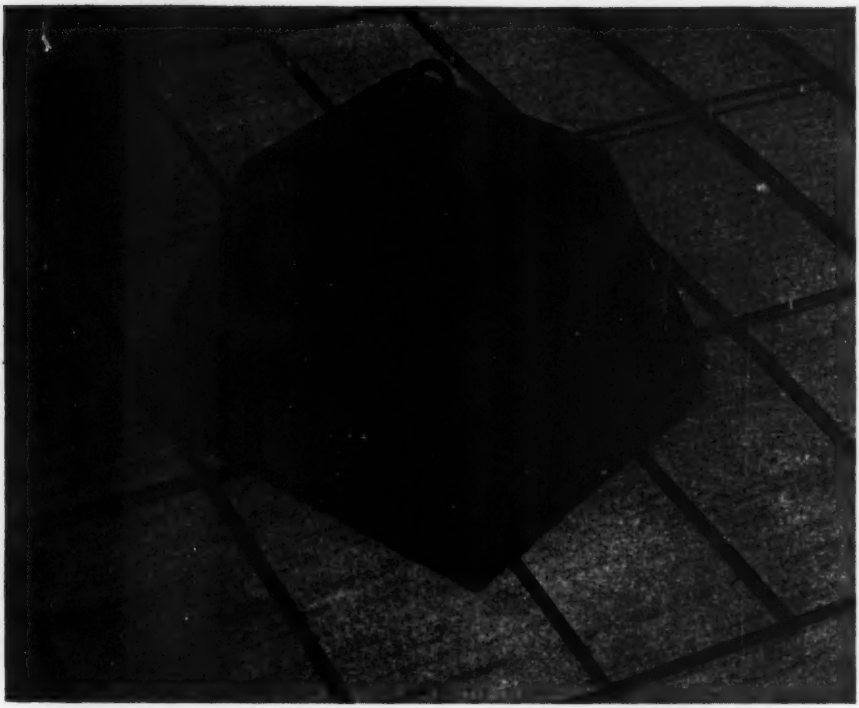
The unit features the Roto-Rol Mixer in top pressure vessel to assure continuous operation; Augermatic Feed provides positive flow control with a continuous feed from a single outlet for smooth, uninterrupted flow of material, wet or dry. Skip loader, operated with simple foot pedal control, screens sand and cement into separate compartments. Other features include built-in water meter variable feed control and large 15 in. fill door with self-cleaning, self-sealing slide valve.—Air Placement Equipment Co., 1012A West 25th St., Kansas City 8, Mo.

Literature Available

Pertinent details on the latest equipment and products on the market are available in recently released literature. Exact titles of the booklets and catalogs are indicated in capital letters. They may be requested directly from the manufacturers listed below.

CYCLOBLOWERS (Bulletin 160)—Four-page bulletin gives performance curves and other technical information on two new Cyclo-Blowers for pneumatic unloading of bulk trailers. With air discharge pressures up to 16 psig in continuous operation, up to 20 psig in intermittent operation, these blowers may be used for unloading cement and similarly dense granular materials.—CycloBlower Co., York, Pa.

BUILT-UP ROOFING MANUAL—The 1961 Barrett Built-Up Roofing Manual, formerly known as the Architects and Engineers Reference Manual, presents basic requirements for trouble-free built-up roofing installations. The 52-page manual covers specifications for each type of bonded roof in detail, discussing fully both pitch and asphalt application on all types of roof deck. The manual depicts many flashing details together with drainage and vent flashing systems and appropriate materials. Included are simplified illustrations and tables which enable architects to select materials appropriate to the type of Barrett Bond required.—Product News Section, Barrett Division, Allied Chemical Corp., 75 West Street, New York 6, N. Y.



Heavy loads shorten concrete road life... CF&I Welded Wire Fabric lengthens it

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SOUTHWESTERN BOOSTS CAPACITY WITH NEW TEXAS PLANT—Reprinted from a national trade magazine, this 8-page booklet describes how Southwestern Portland Cement Co. uses an x-ray spectrograph, the Norelco Autrometer, to maintain hourly control of calcium, iron, and alumina in the raw mix at its new Odessa plant. Provides facts on the construction and facilities of the Odessa plants.—Philips Electronic Instruments, 750 South Fulton Ave., Mount Vernon, N.Y.

BRIDGE CONCRETE PROBLEMS (MBR-P-11)—Case history reports of more than 200 bridge concreting problems and how they were solved are studied in this 16-page publication. Included are reports on concreting of piers and bridge decks for highway and railway bridges, hot and cold weather concreting data, the use of lightweight aggregate in bridge work, and placing and finishing problems encountered in bridge work, setting forth the part played by Pozzolith concrete in meeting various conditions.—The Master Builders Co., Cleveland 18, Ohio

BLAST RESISTANT DESIGN

DESIGN OF BLAST RESISTANT CONSTRUCTION FOR ATOMIC EXPLOSIONS. This is a corrected version of a paper which presents methods and principles used in designing the first full scale blast resistant structures tested in Eniwetok in the Pacific. Test results are cited in support of the procedures outlined. Also discussed are economic and other practical considerations; and radiation hazards and methods of dealing with them. Appendices detail procedures for computing the blast loading, for designing individual structural elements and single and multistory buildings in both the elastic and plastic range for this loading, and for computing ultimate strength of structural elements and frames under rapid loading. 94 pp. Price $1.75.

AMERICAN CONCRETE INSTITUTE PUBLICATIONS

THE STORY OF CONCRETE PRESSURE PIPE—The principles of prestressing and how prestressed pipe is manufactured is explained in this well illustrated, 24-page booklet. A complete table of sizes, weights and normal operating heads for each type of pipe used for water transmission and distribution lines, subaqueous intakes and outfalls, and sewage force mains is included.—Price Brothers Co., 1932 E. Monument Ave., Dayton 1, Ohio

TOOL FOR MINIATURE MEASUREMENTS (TB1A)—Pocket model magnifying comparator designed for greater accuracy when making miniature measurements. The instrument is said to be ideal for crack measurement and is described as an unbreakable tool with optical precision; accurate, versatile, and easy to use. Literature describes pocket and desk models.—Finescale Co., 218 South Western Ave., Los Angeles 4, Calif.

TAPPING PRESTRESSED CONCRETE STEEL CYLINDER PIPE UNDER PRESSURE—Handy pocket-size, 12-page booklet describes the step-by-step procedure for tapping prestressed concrete steel cylinder pipe under pressure. The manual discusses the basic construction of the pipe, and describes how small and large taps can be made quickly and safely with ordinary tapping tools and with no interruption in service.—Price Brothers Co., 1932 E. Monument Ave., Dayton 1, Ohio

HIGH STRENGTH FINE GRAINED CARBON STEELS—Brochure gives the composition and physical properties of GLX-W columbium-treated steels. Because of their low carbon content, the bars are said to be weldable without special precautions as to technique and electrodes. The bar can be bent flat on itself and will have yield strengths as specified up to 60,000 psi minimum. The steel is reasonably priced so that the reduction in diameter possible will result in monetary savings according to company spokesman.—Great Lakes Steel Corp., Division of National Steel Corp., Ecorse, Detroit 29, Mich.

AUTOCLAVING SYSTEM FOR CURING CONCRETE BLOCKS (SW 161)—Eight-page folder describes the Petrotherm System, a new autoclave development which was designed especially for curing concrete blocks. Using a fired heater instead of a boiler or steam generator, the system is reported to provide the ultimate in high pressure curing. Brochure includes a typical layout of the Petrotherm System and details on its use and advantages; various components are illustrated and described.—Struthers Wells Corp., Special Equipment Division, Titusville, Pa.

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